Deep Learning II
Unsupervised Learning

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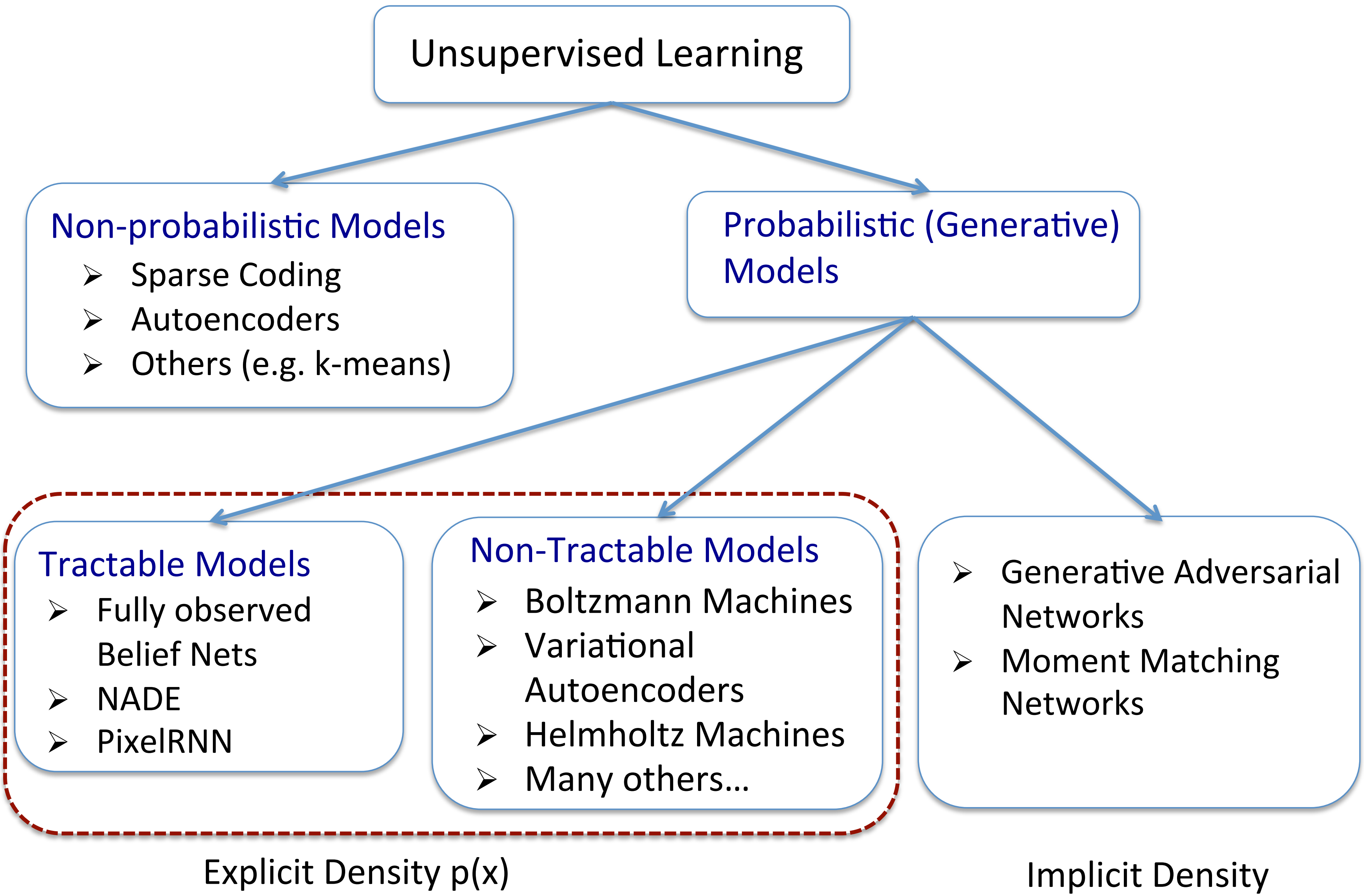
CANADIAN INSTITUTE
for ADVANCED RESEARCH

Talk Roadmap

Part 1: Supervised Learning: Deep Networks

Part 2: Unsupervised Learning: Learning Deep Generative Models

Part 3: Open Research Questions


Unsupervised Learning
Non-probabilistic Models
Sparse Coding
Autoencoders
Others (e.g. k-means)
Probabilistic (Generative) Models
Tractable Models
Fully observed Belief Nets
NADE
PixelRNN
Non-Tractable Models
Boltzmann Machines
Variational Autoencoders
Helmholtz Machines
Many others…
Generative Adversarial Networks
Moment Matching Networks
Explicit Density p(x)
Implicit Density

Talk Roadmap

- Basic Building Blocks:
  - ➢ Sparse Coding
  - ➢ Autoencoders
- Deep Generative Models
  - ➢ Restricted Boltzmann Machines
  - ➢ Deep Belief Networks and Deep Boltzmann Machines
  - ➢ Helmholtz Machines / Variational Autoencoders
- Generative Adversarial Networks
- Model Evaluation

Sparse Coding

- Sparse coding (Olshausen & Field, 1996). Originally developed to explain early visual processing in the brain (edge detection).

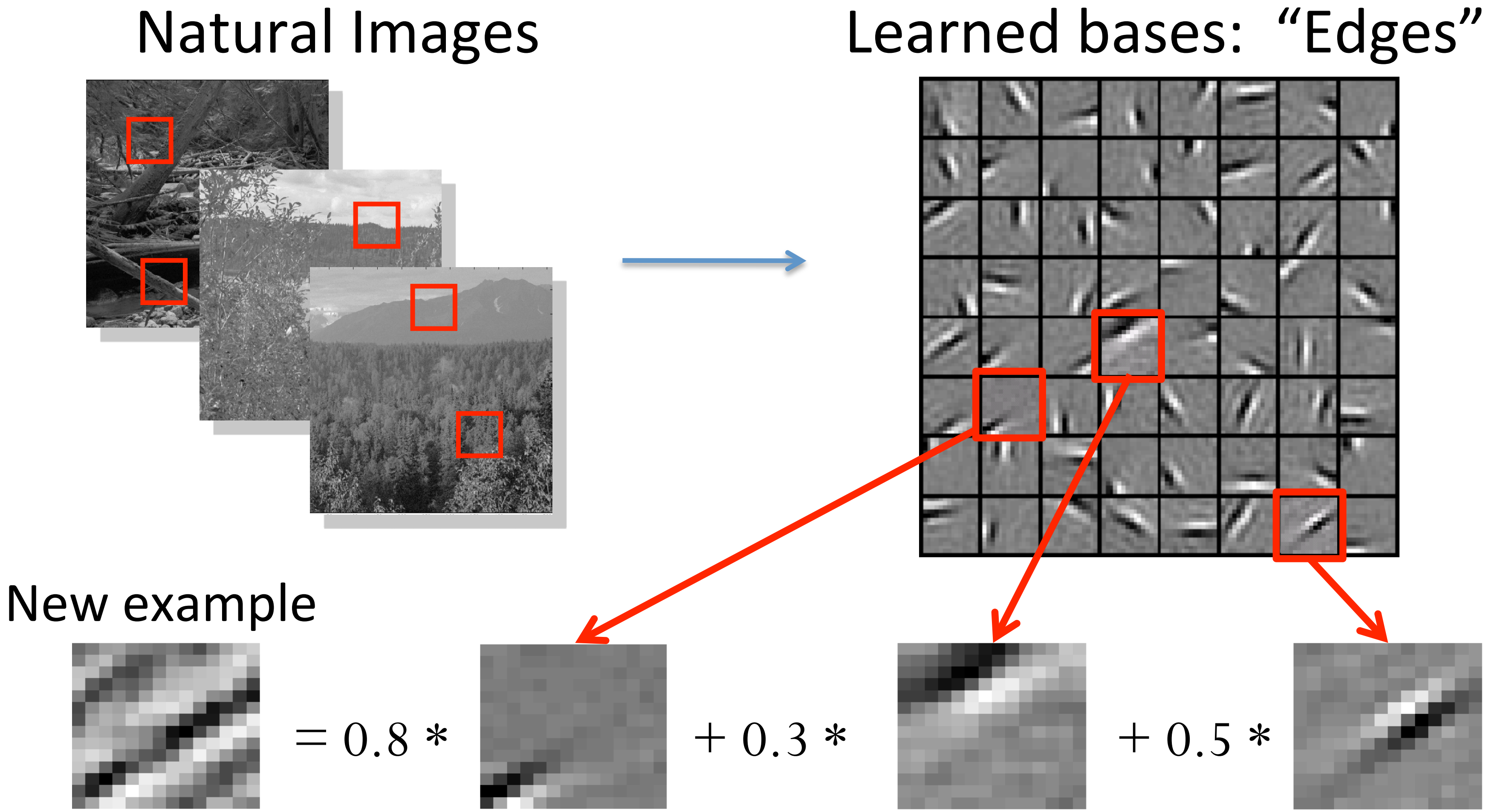
- Objective: Given a set of input data vectors $\{\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N\}$, learn a dictionary of bases $\{\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, ..., \boldsymbol{\phi}_K\}$, such that:

$$\mathbf{x}_n = \sum_{k=1}^{K} a_{nk} \boldsymbol{\phi}_k,$$

Sparse: mostly zeros

- Each data vector is represented as a sparse linear combination of bases.

Sparse Coding



$$x = 0.8 * \phi_{36} + 0.3 * \phi_{42} + 0.5 * \phi_{65}$$

[0, 0, … **0.8**, …, **0.3**, …, **0.5**, …] = coefficients (feature representation)

Slide Credit: Honglak Lee

Sparse Coding: Training

- Input image patches: $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N \in \mathbb{R}^D$
- Learn dictionary of bases: $\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, ..., \boldsymbol{\phi}_K \in \mathbb{R}^D$

$$\min_{\mathbf{a},\boldsymbol{\phi}} \underbrace{\sum_{n=1}^{N} \left\| \mathbf{x}_n - \sum_{k=1}^{K} a_{nk} \boldsymbol{\phi}_k \right\|_2^2}_{\text{Reconstruction error}} + \underbrace{\lambda \sum_{n=1}^{N} \sum_{k=1}^{K} |a_{nk}|}_{\text{Sparsity penalty}}$$

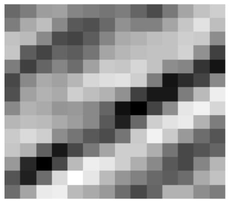
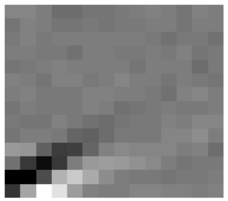
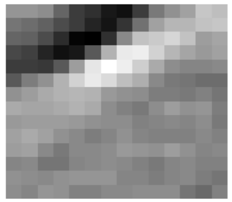
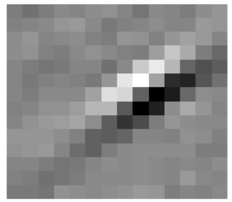
- Alternating Optimization:

1. Fix dictionary of bases $\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, ..., \boldsymbol{\phi}_K$ and solve for activations **a** (a standard Lasso problem).
2. Fix activations **a**, optimize the dictionary of bases (convex QP problem).

Sparse Coding: Testing Time

- Input: a new image patch x* , and K learned bases $\phi_1, \phi_2, ..., \phi_K$
- Output: sparse representation **a** of an image patch x*.

$$\min_{\mathbf{a}} \left\| \mathbf{x}^* - \sum_{k=1}^{K} a_k \phi_k \right\|_2^2 + \lambda \sum_{k=1}^{K} |a_k|$$

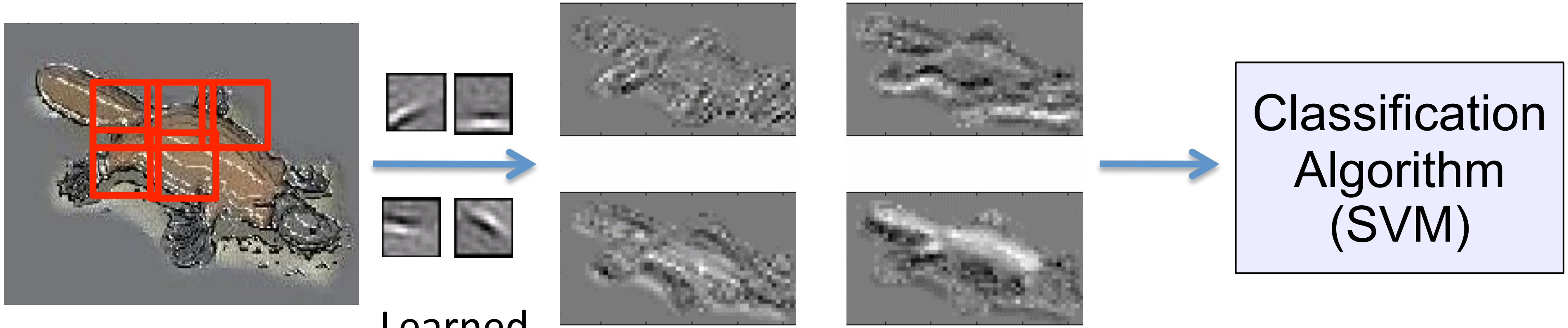
 $= 0.8 *$  $+ 0.3 *$  $+ 0.5 *$ 

$x^* = 0.8 * \phi_{36} + 0.3 * \phi_{42} + 0.5 * \phi_{65}$

[0, 0, … **0.8**, …, **0.3**, …, **0.5**, …] = coefficients (feature representation)

Image Classification

Evaluated on Caltech101 object category dataset.

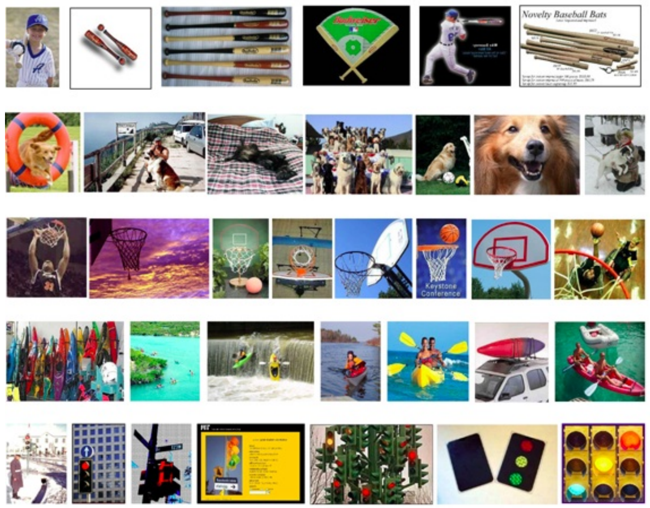


Input Image

Learned bases

Features (coefficients)

9K images, 101 classes



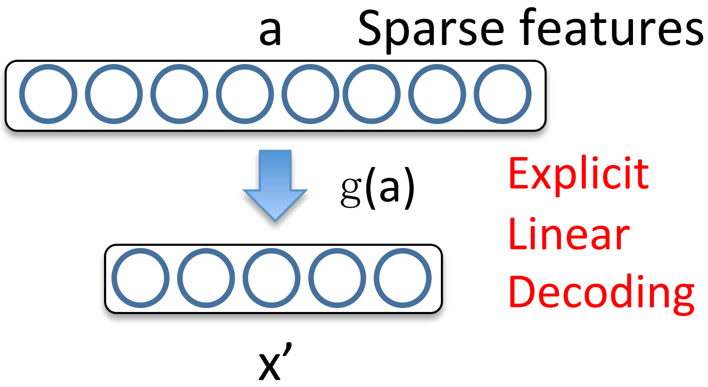
Algorithm	Accuracy
Baseline (Fei-Fei et al., 2004)	16%
PCA	37%
Sparse Coding	**47%**

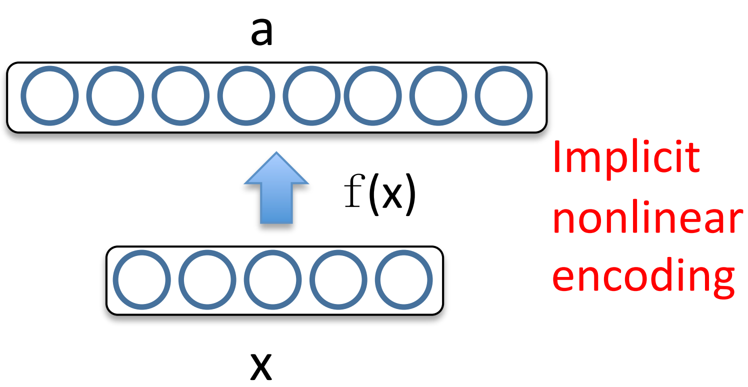
Slide Credit: Honglak Lee

(Lee, Battle, Raina, Ng, NIPS 2007)

Interpreting Sparse Coding

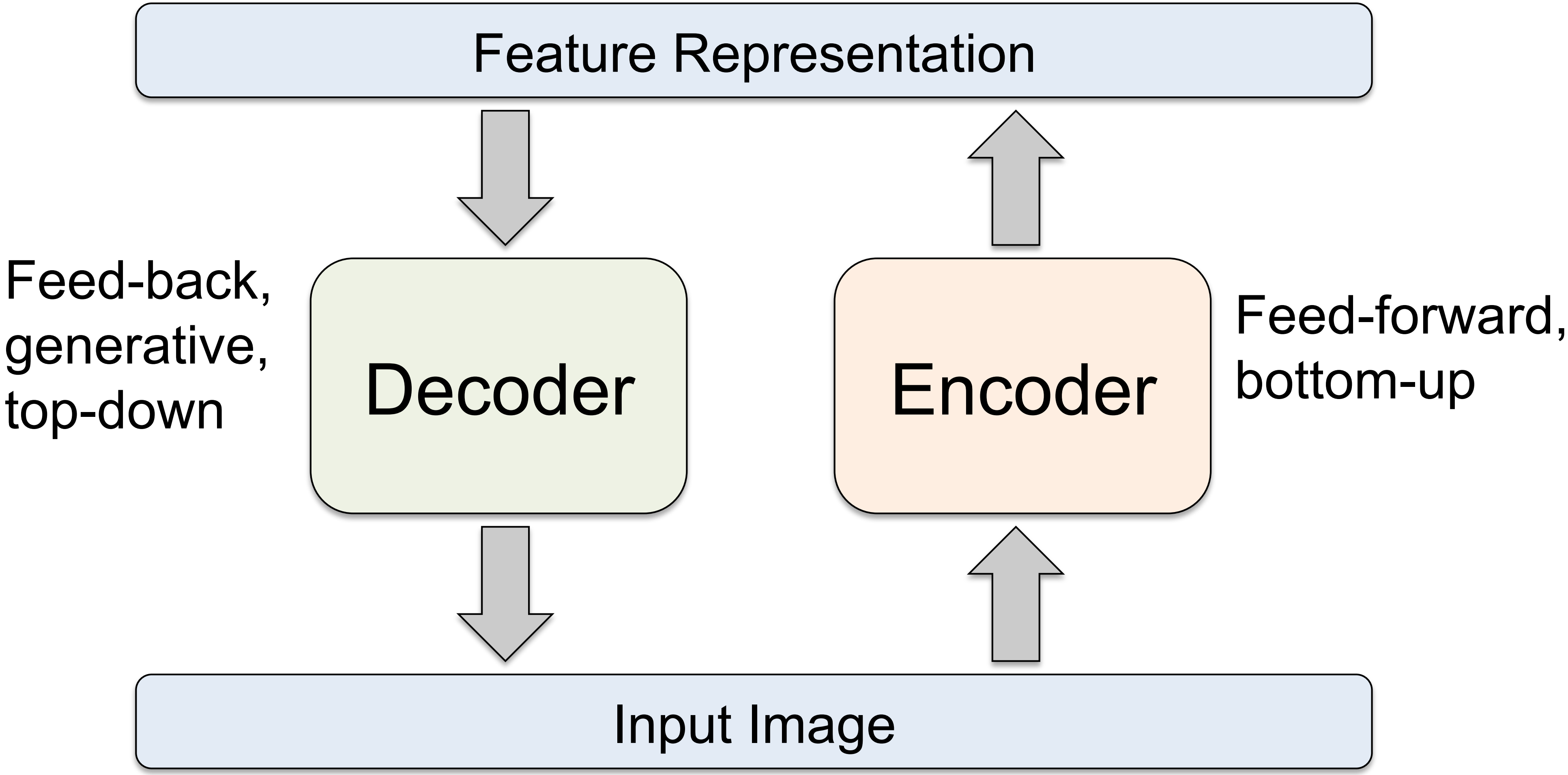
$$\min_{\mathbf{a},\phi} \sum_{n=1}^{N} \left\| \mathbf{x}_n - \sum_{k=1}^{K} a_{nk}\phi_k \right\|_2^2 + \lambda \sum_{n=1}^{N} \sum_{k=1}^{K} |a_{nk}|$$





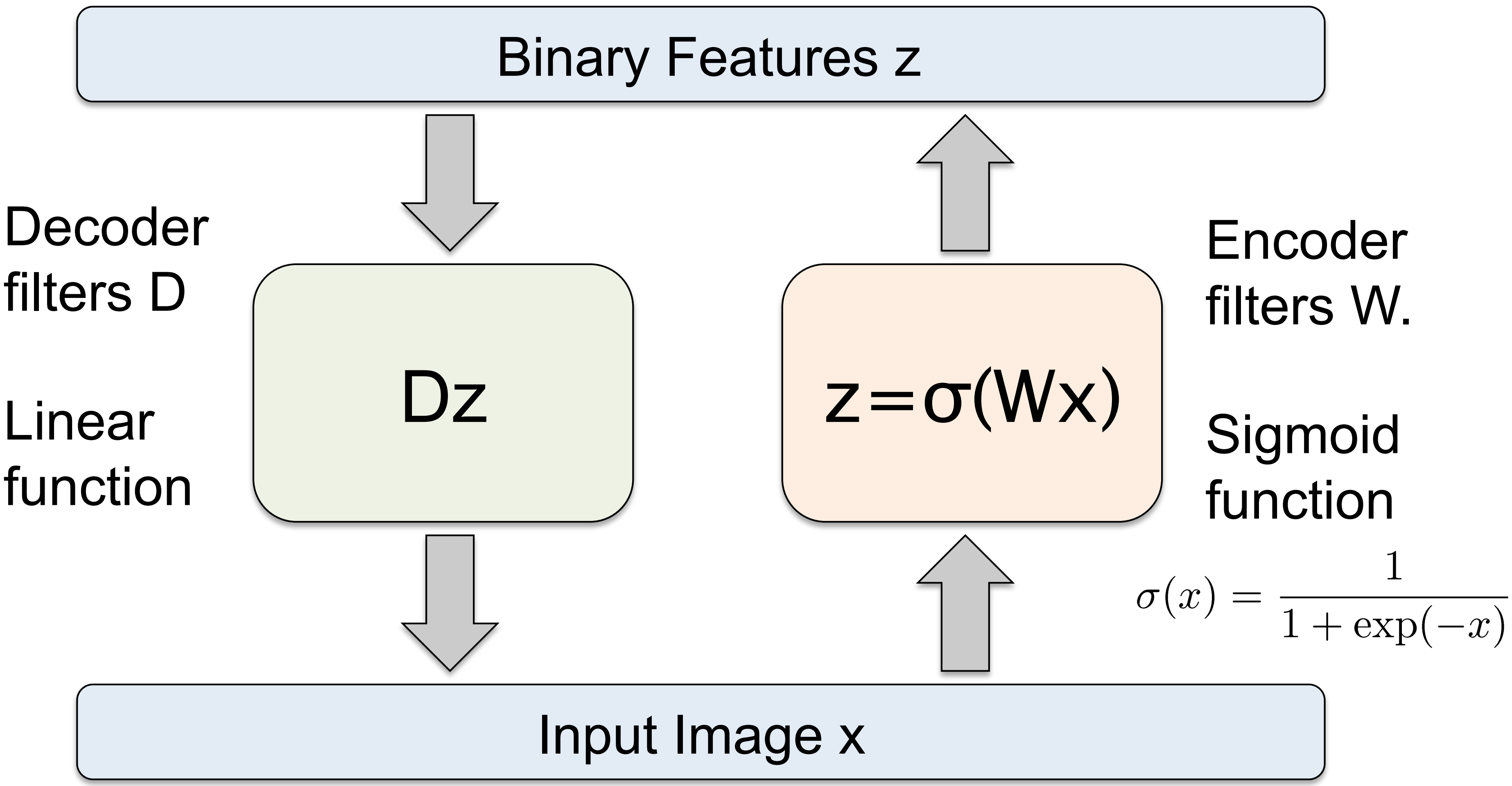
- Sparse, over-complete representation **a.**
- Encoding **a** = f(**x**) is implicit and nonlinear function of **x**.
- Reconstruction (or decoding) **x’** = g(**a**) is linear and explicit.

Autoencoder

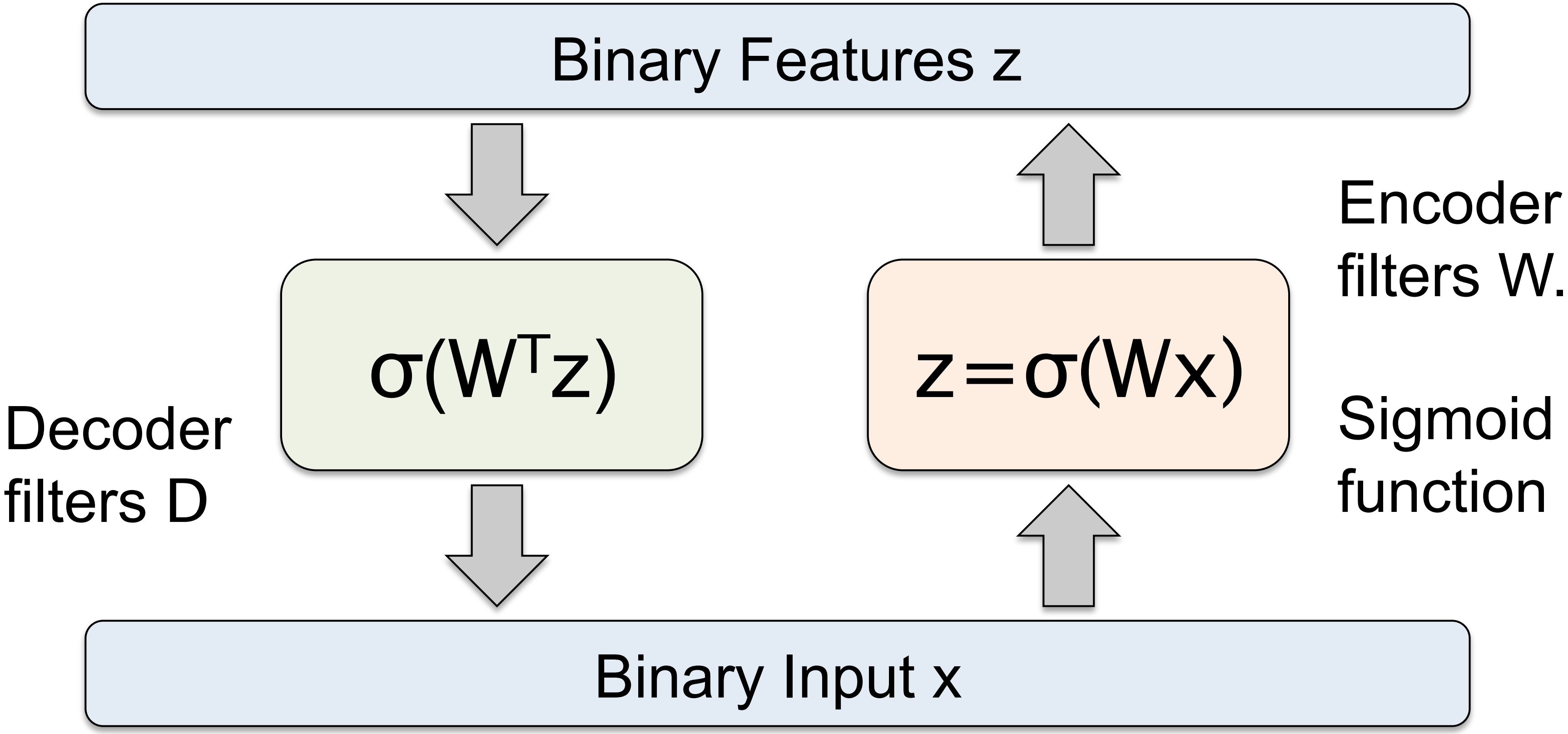


- Details of what goes insider the encoder and decoder matter!
- Need constraints to avoid learning an identity.

Autoencoder



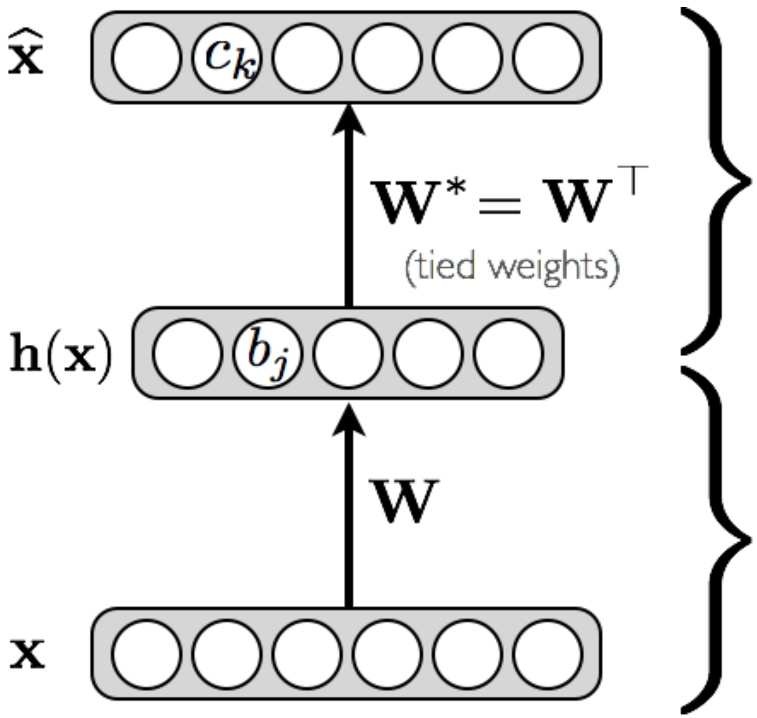
Autoencoder



- Need additional constraints to avoid learning an identity.
- Relates to Restricted Boltzmann Machines (later).

Autoencoders

- Feed-forward neural network trained to reproduce its input at the output layer



Decoder

$$
\begin{aligned}
\widehat{\mathbf{x}} &= o(\widehat{\mathbf{a}}(\mathbf{x})) \\
&= \underbrace{\mathrm{sigm}}_{\text{For binary units}}(\mathbf{c} + \mathbf{W}^* \mathbf{h}(\mathbf{x}))
\end{aligned}
$$

Encoder

$$
\begin{aligned}
\mathbf{h}(\mathbf{x}) &= g(\mathbf{a}(\mathbf{x})) \\
&= \mathrm{sigm}(\mathbf{b} + \mathbf{W}\mathbf{x})
\end{aligned}
$$

Loss Function

- Loss function for binary inputs

$$l(f(\mathbf{x})) = -\sum_k \left(x_k \log(\widehat{x}_k) + (1 - x_k) \log(1 - \widehat{x}_k)\right)$$

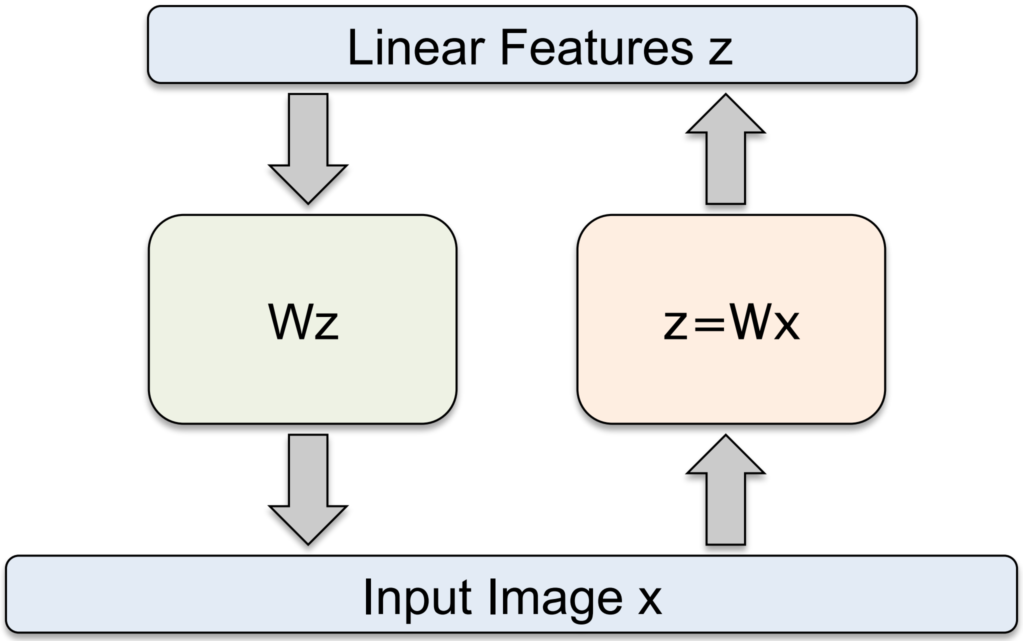
  - ➢ Cross-entropy error function (reconstruction loss) $f(\mathbf{x}) \equiv \widehat{\mathbf{x}}$

- Loss function for real-valued inputs

$$l(f(\mathbf{x})) = \frac{1}{2} \sum_k (\widehat{x}_k - x_k)^2$$

  - ➢ sum of squared differences (reconstruction loss)
  - ➢ we use a linear activation function at the output

Autoencoder



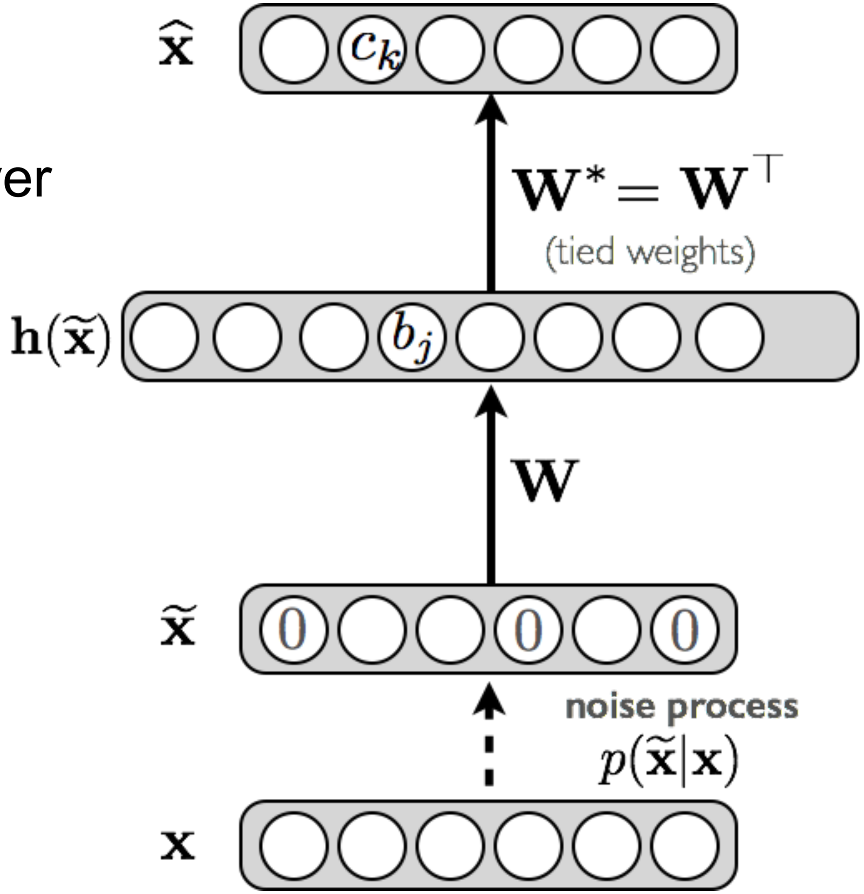
- If the hidden and output layers are linear, it will learn hidden units that are a linear function of the data and minimize the squared error.

- The K hidden units will span the same space as the first k principal components. The weight vectors may not be orthogonal.

- With nonlinear hidden units, we have a nonlinear generalization of PCA.

Denoising Autoencoder

- Idea: representation should be robust to introduction of noise:
  - random assignment of subset of inputs to 0, with probability ν
  - Similar to dropouts on the input layer
  - Gaussian additive noise



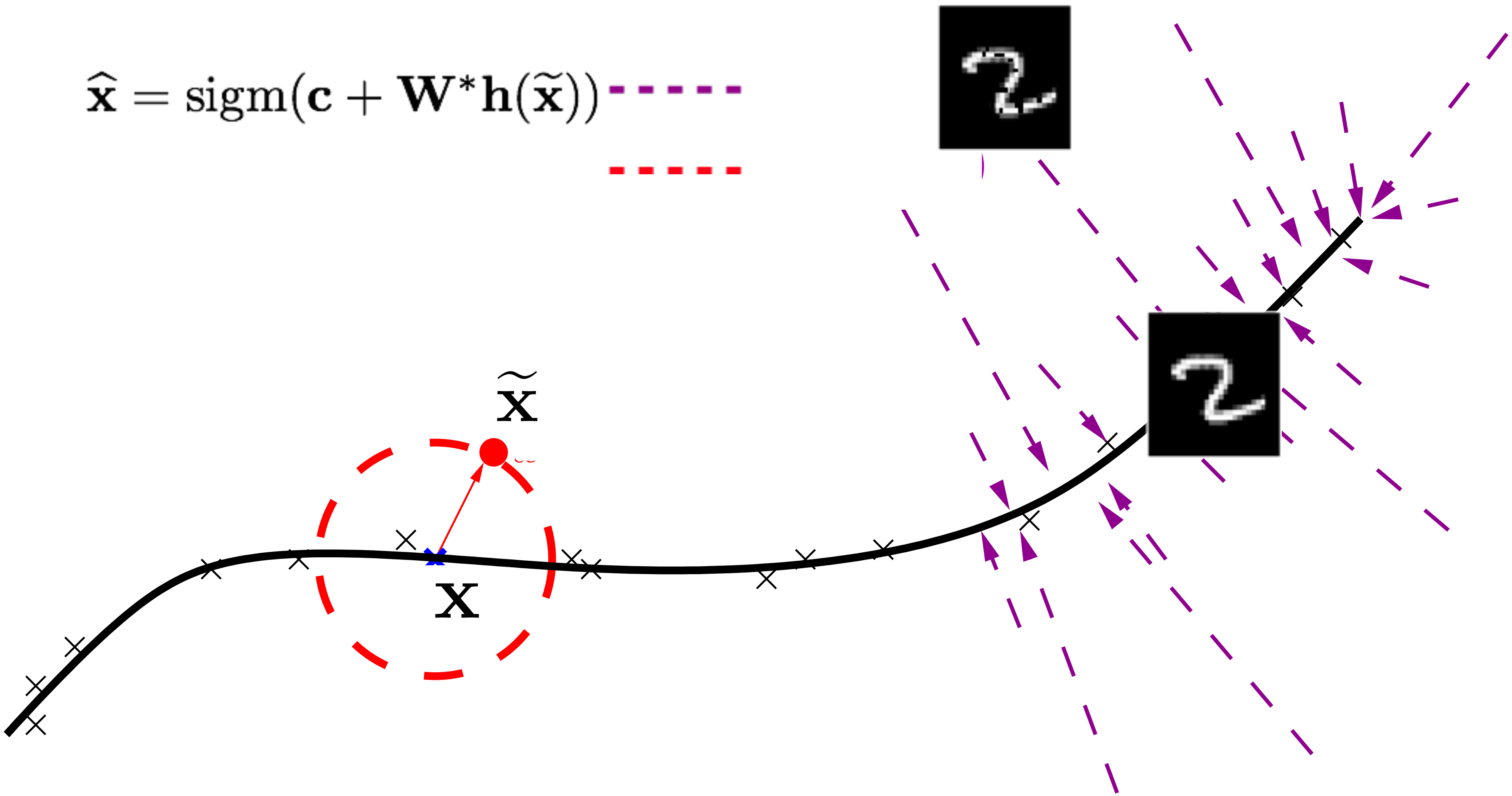
- Reconstruction $\widehat{\mathbf{x}}$ computed from the corrupted input $\widetilde{\mathbf{x}}$

- Loss function compares $\widehat{\mathbf{x}}$ reconstruction with the noiseless input $\mathbf{x}$

(Vincent et al., ICML 2008)

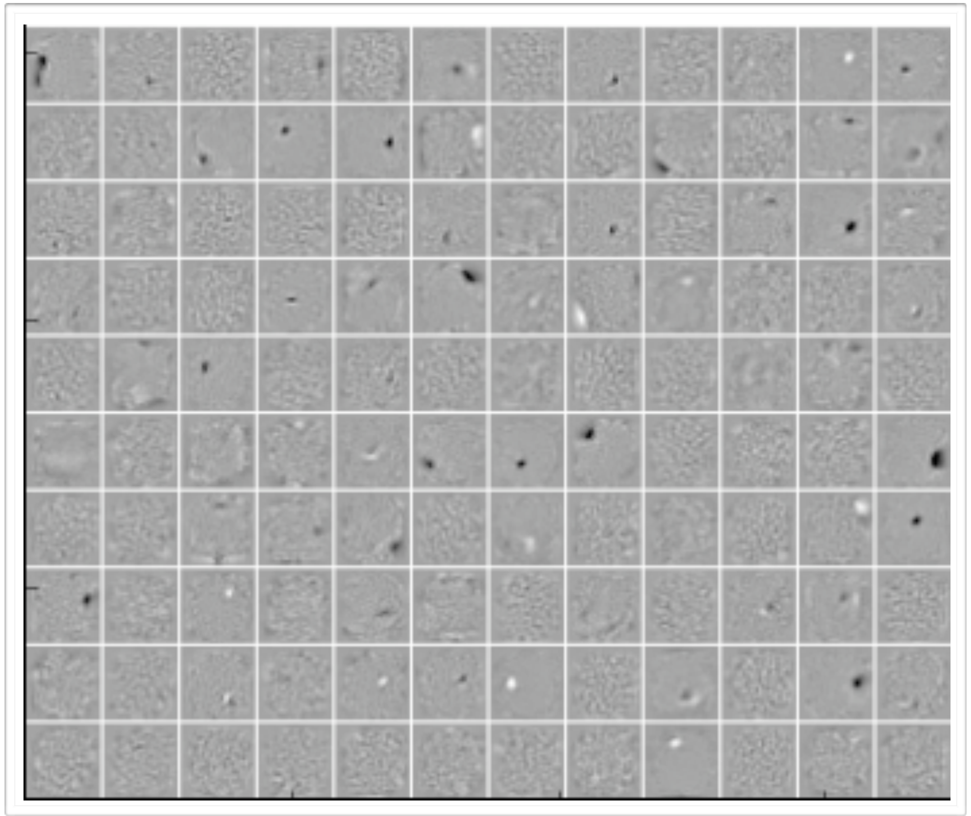
Denoising Autoencoder

$$\widehat{\mathbf{x}} = \mathrm{sigm}(\mathbf{c} + \mathbf{W}^* \mathbf{h}(\widetilde{\mathbf{x}}))$$

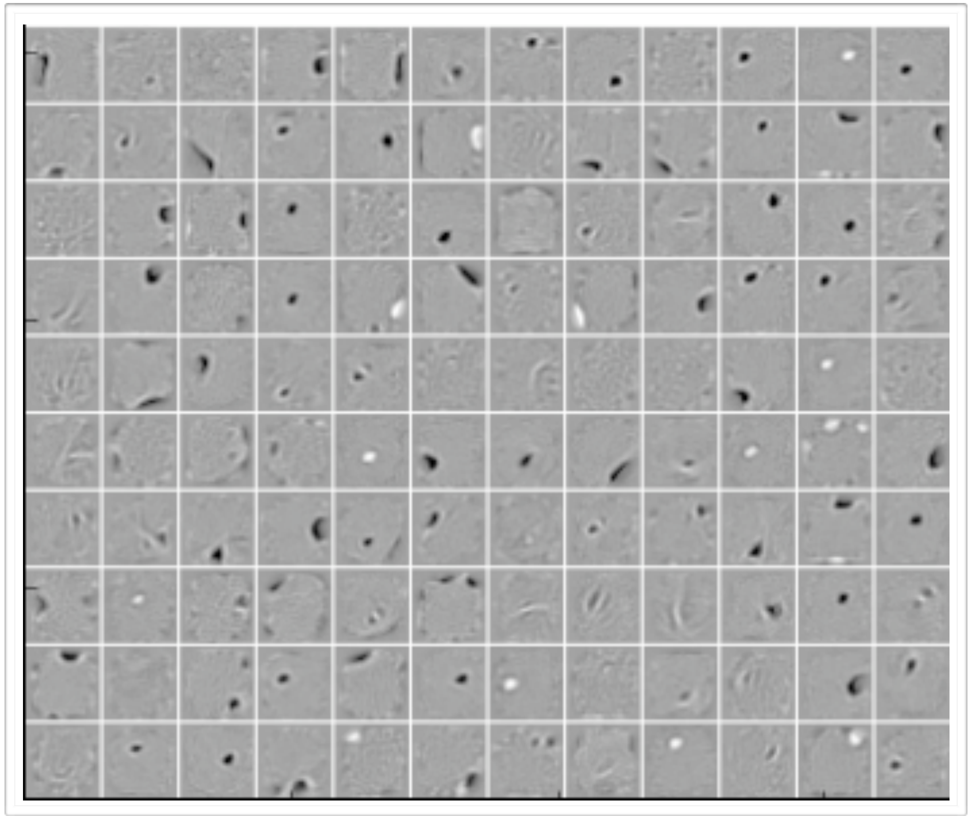


Learned Filters

Non-corrupted



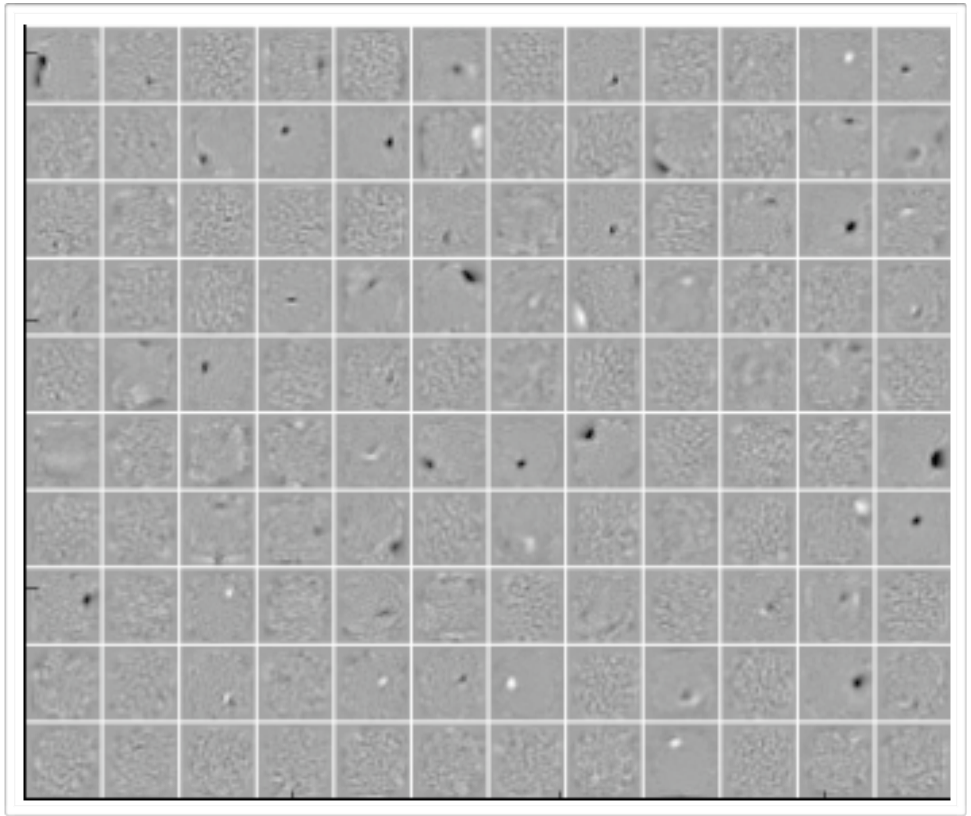
25% corrupted input



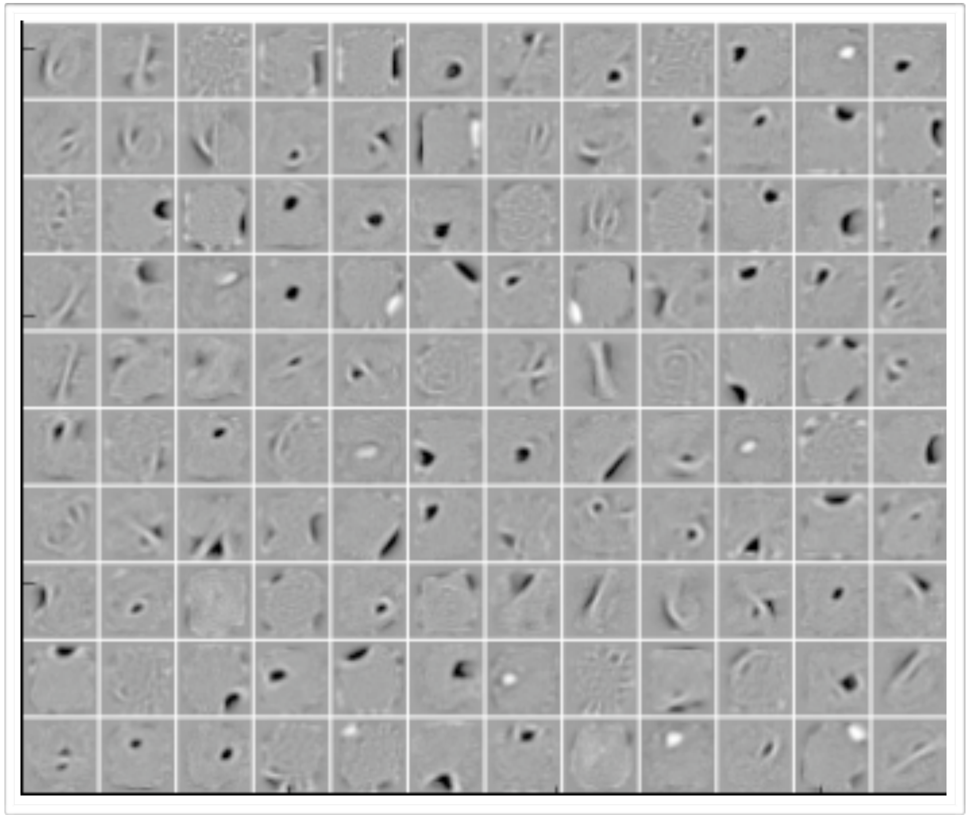
(Vincent et al., ICML 2008)

Learned Filters

Non-corrupted

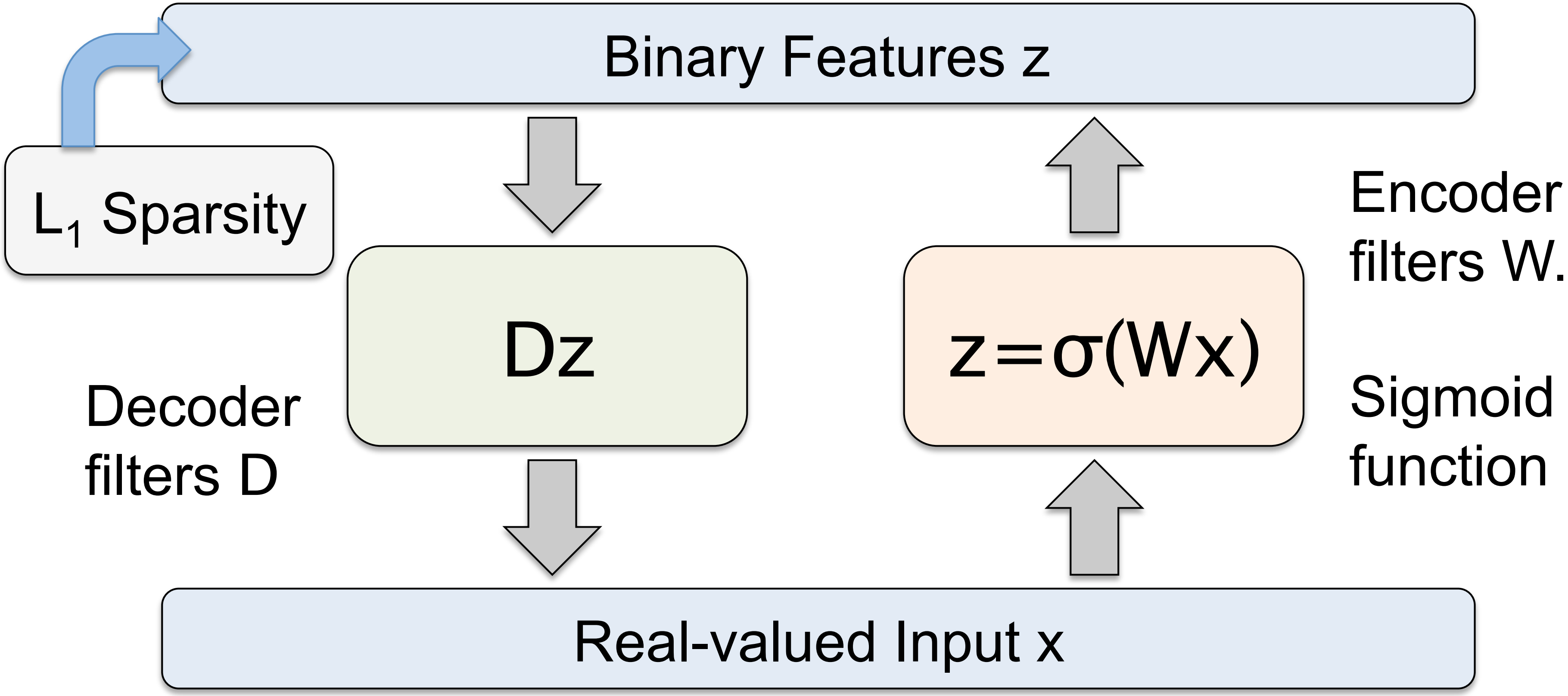


50% corrupted input



(Vincent et al., ICML 2008)

Predictive Sparse Decomposition

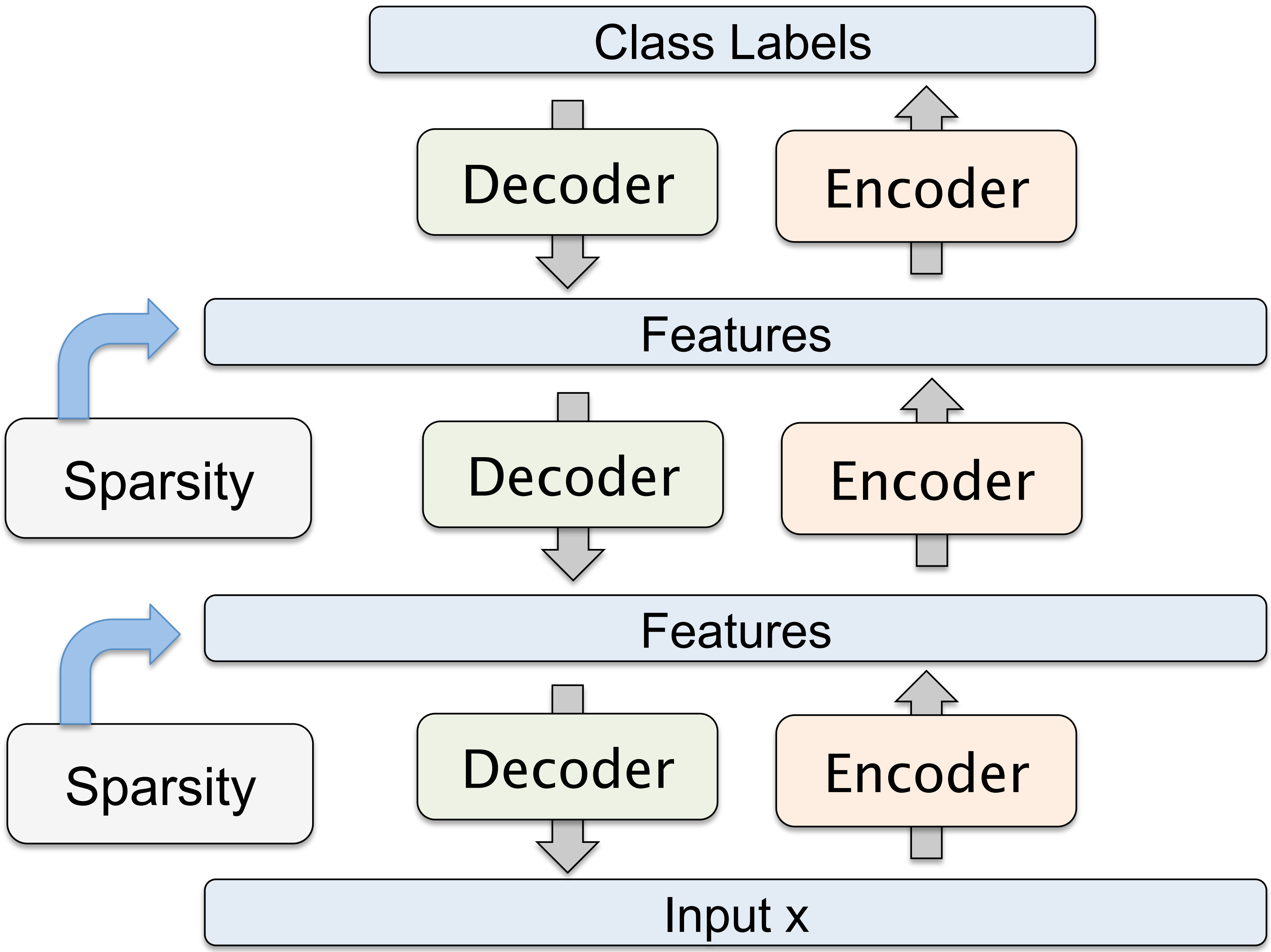


At training time

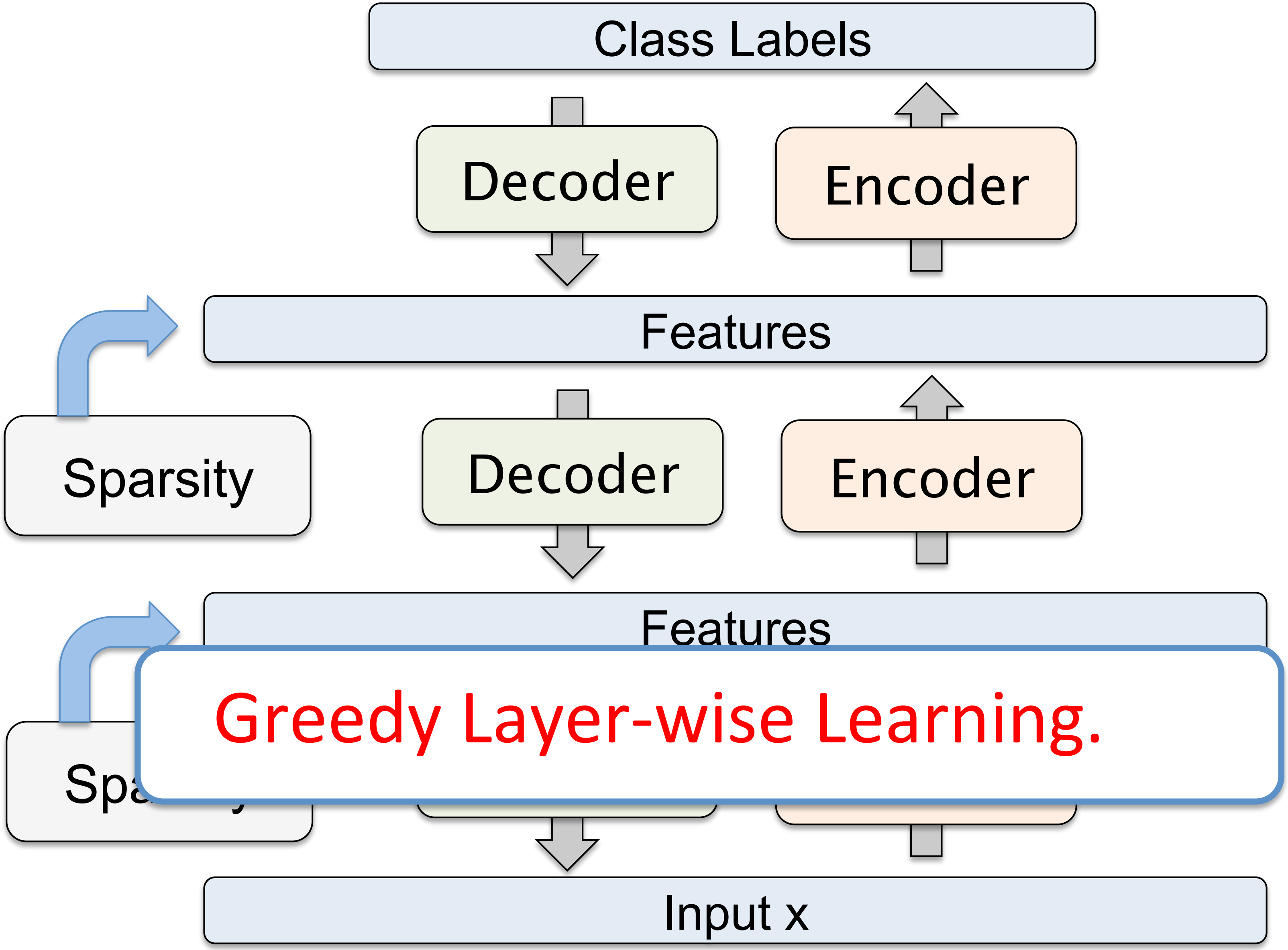
$$\min_{D,W,\mathbf{z}} \underbrace{||D\mathbf{z}-\mathbf{x}||_2^2 + \lambda|\mathbf{z}|_1}_{\text{Decoder}} + \underbrace{||\sigma(W\mathbf{x})-\mathbf{z}||_2^2}_{\text{Encoder}}$$

(Kavukcuoglu, Ranzato, Fergus, LeCun, 2009)

Stacked Autoencoders



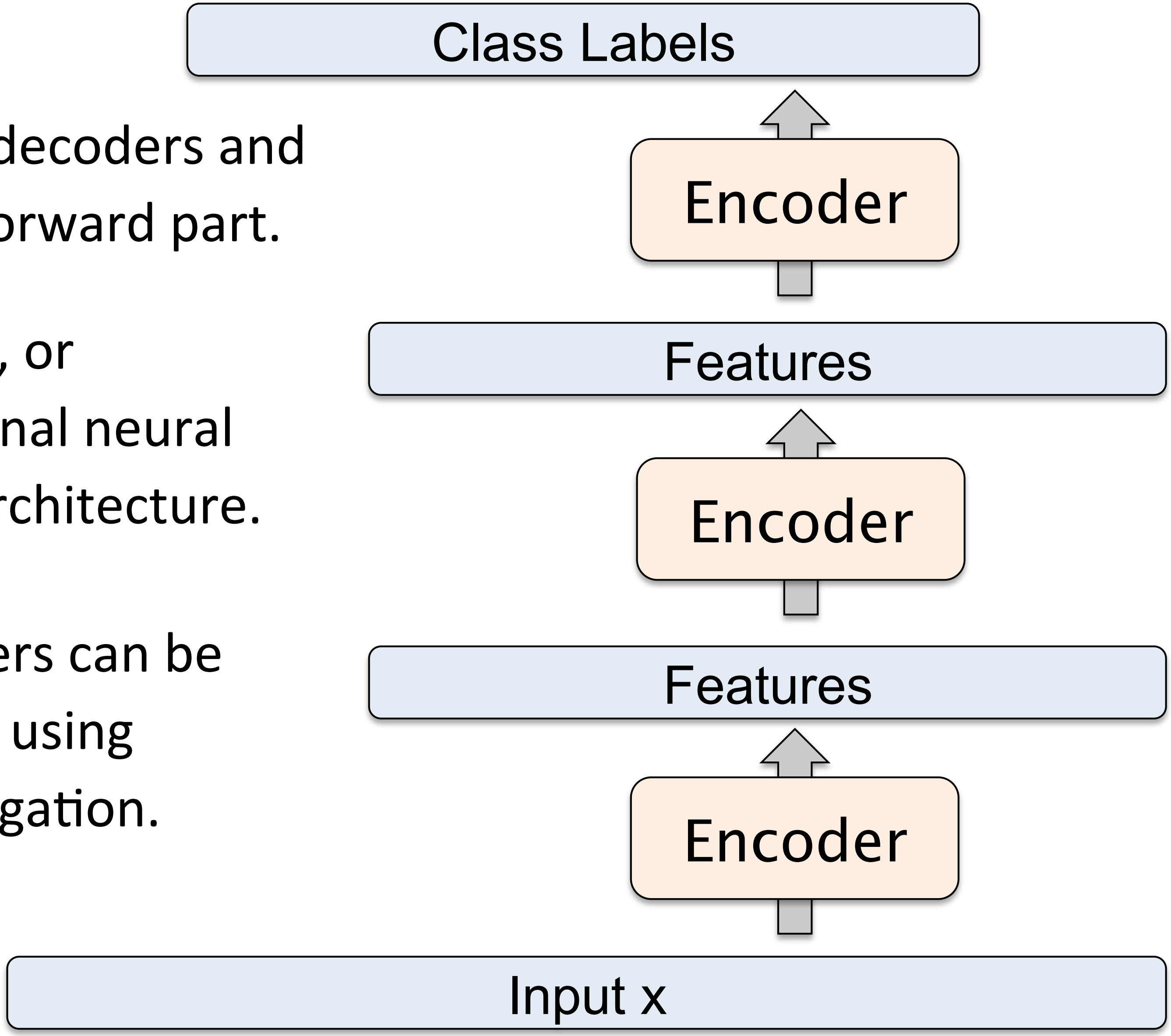
Stacked Autoencoders



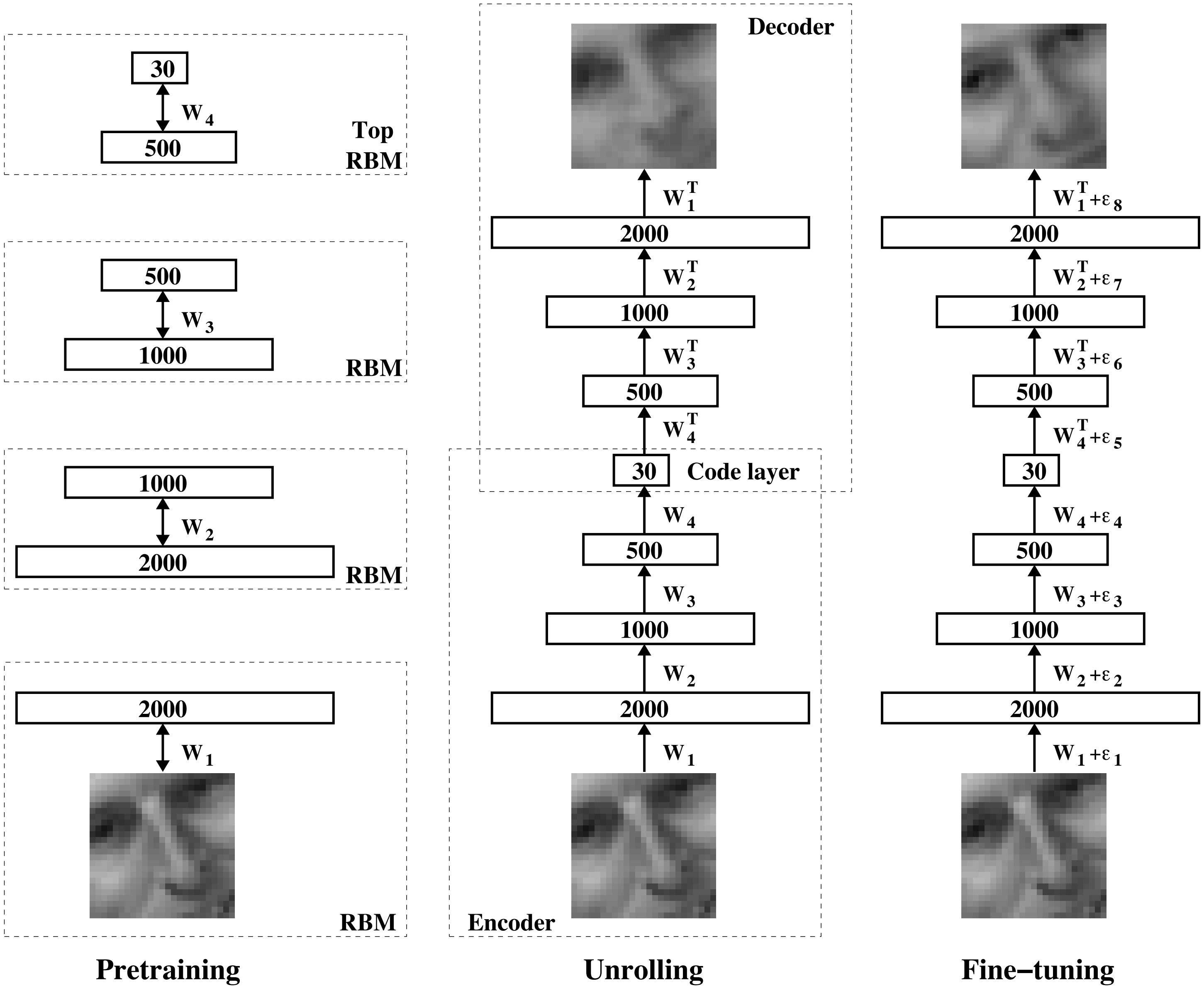
Greedy Layer-wise Learning.

Stacked Autoencoders

- Remove decoders and use feed-forward part.
- Standard, or convolutional neural network architecture.
- Parameters can be fine-tuned using backpropagation.



Deep Autoencoders



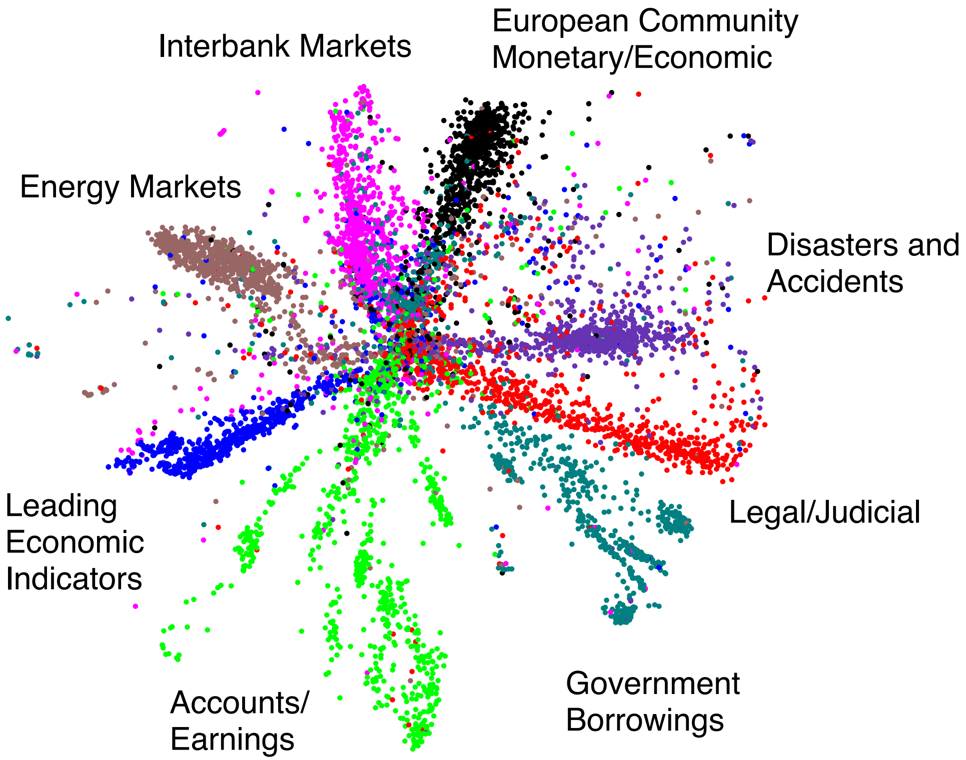
Deep Autoencoders

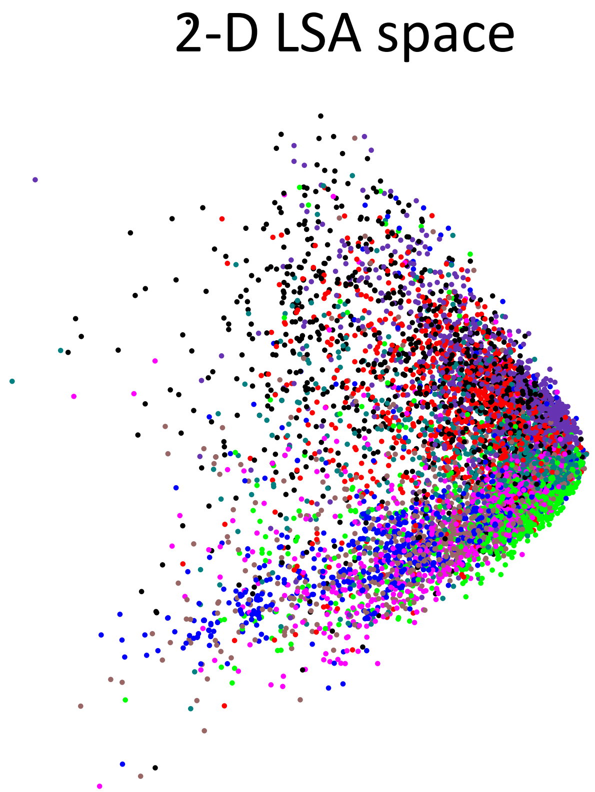
- 25x25 – 2000 – 1000 – 500 – 30 autoencoder to extract 30-D real-valued codes for Olivetti face patches.

- **Top**: Random samples from the test dataset.
- **Middle**: Reconstructions by the 30-dimensional deep autoencoder.
- **Bottom**: Reconstructions by the 30-dimentinoal PCA.

(Hinton and Salakhutdinov, Science 2006)

Information Retrieval



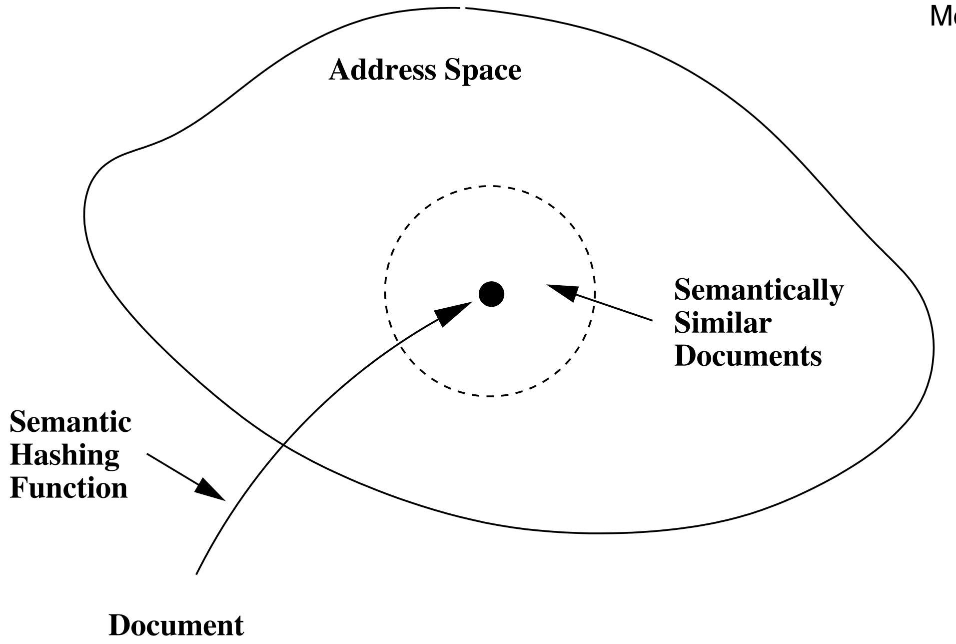


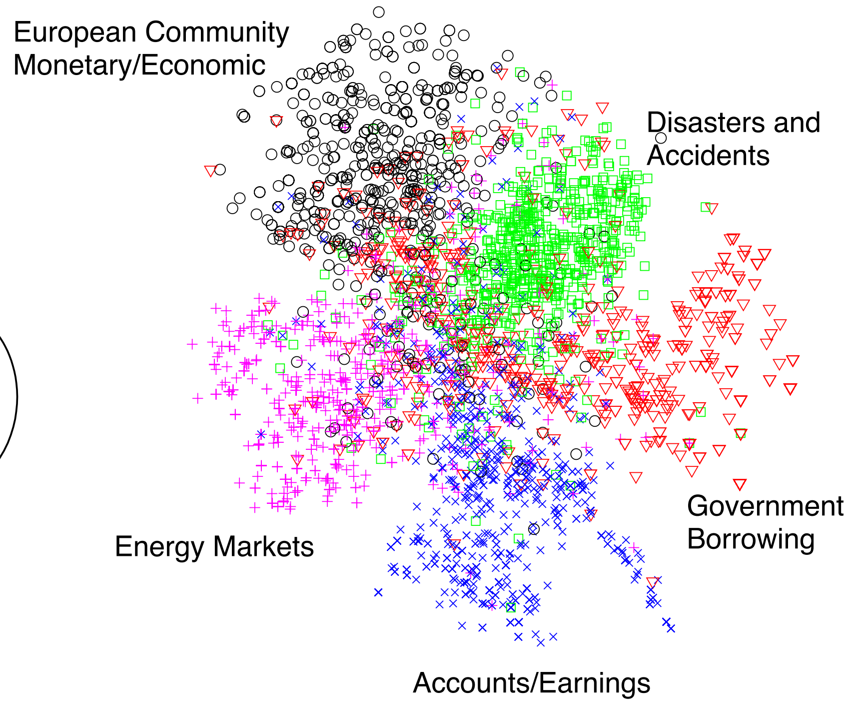
• The Reuters Corpus Volume II contains 804,414 newswire stories (randomly split into **402,207 training** and **402,207 test).**

• "Bag-of-words" representation: each article is represented as a vector containing the counts of the most frequently used 2000 words in the training set.

(Hinton and Salakhutdinov, Science 2006)

Semantic Hashing



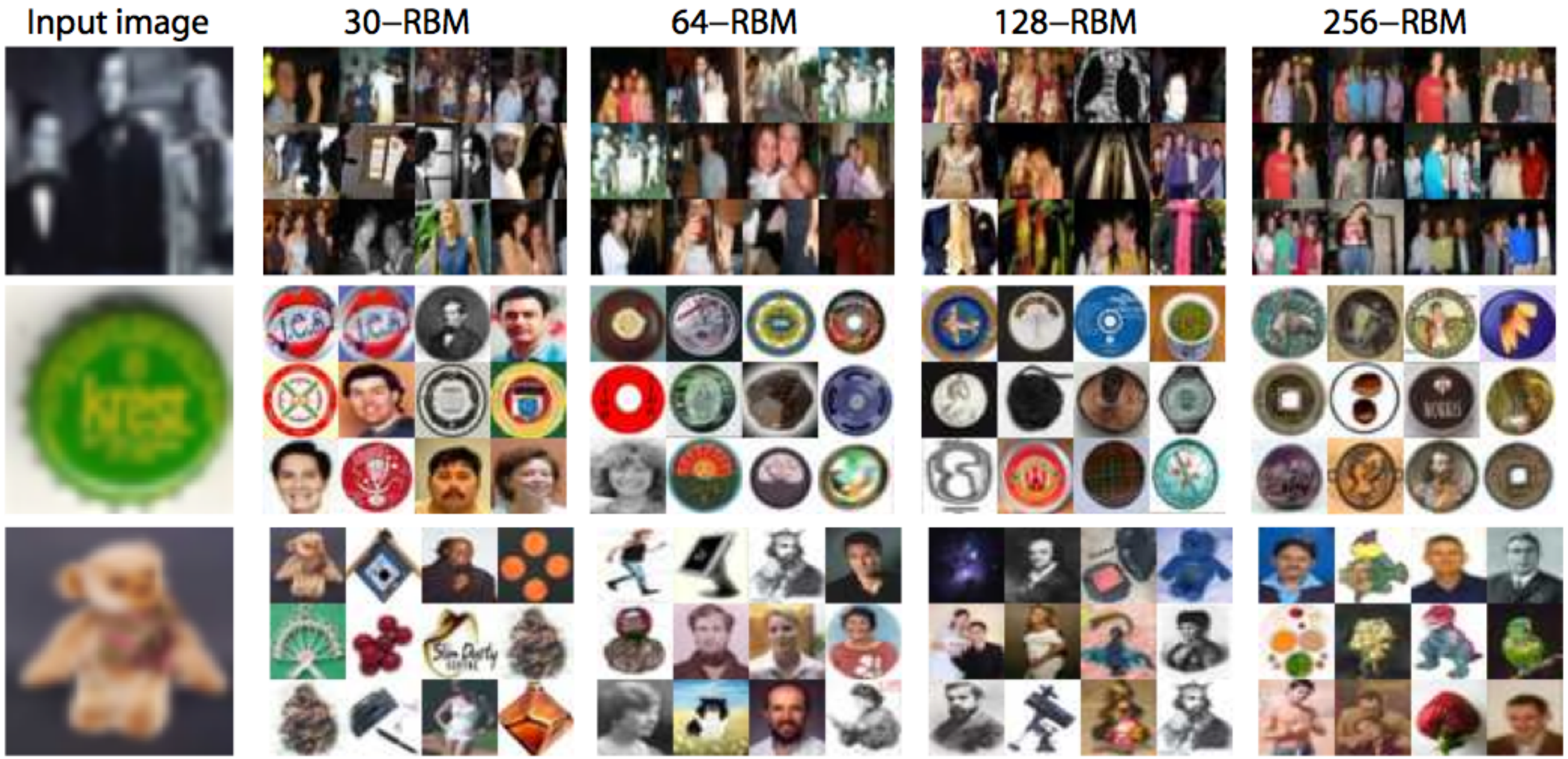


- Learn to map documents into **semantic 20-D binary codes.**
- Retrieve similar documents stored at the nearby addresses **with no search at all.**

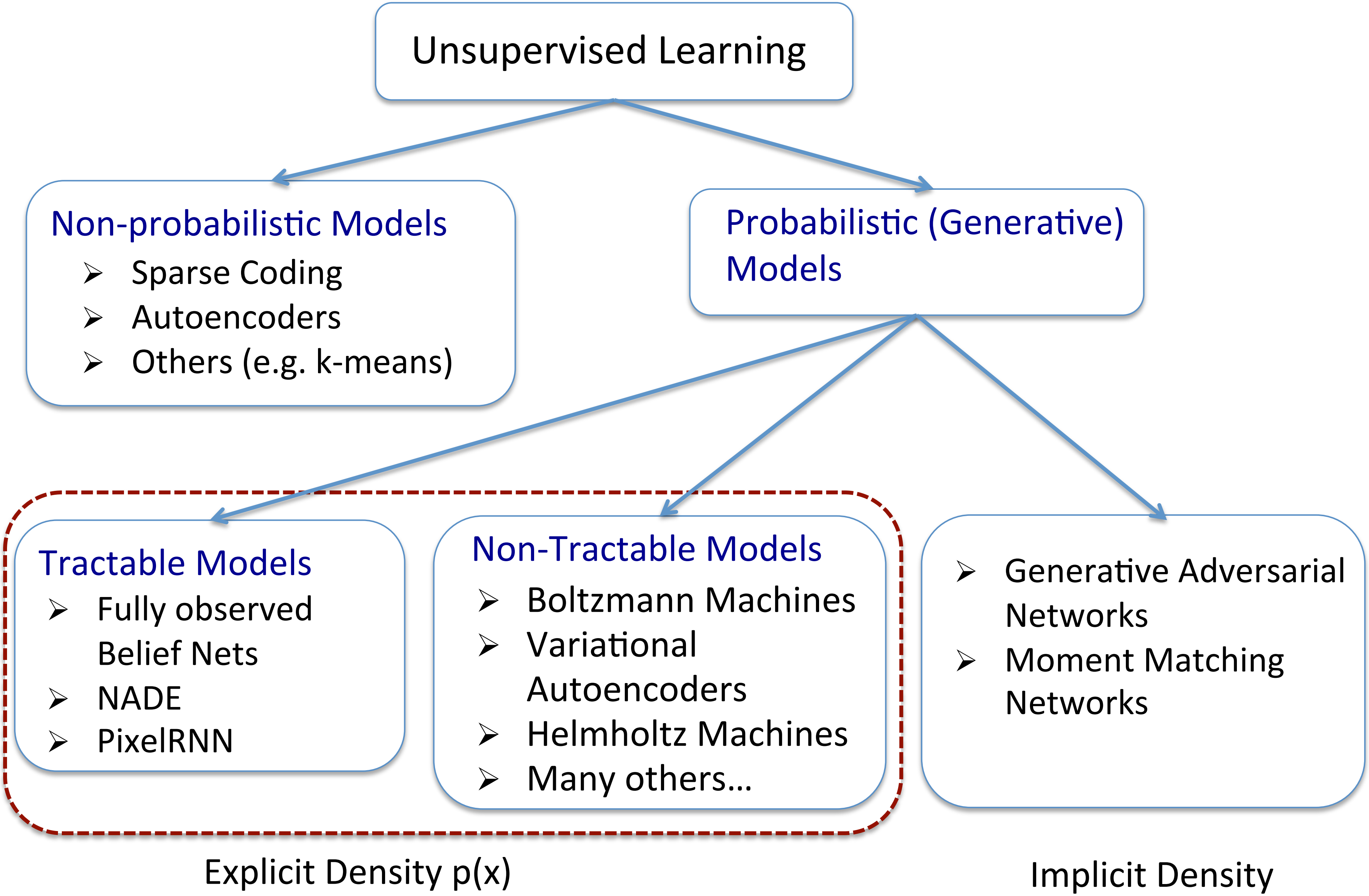
(Salakhutdinov and Hinton, SIGIR 2007)

Searching Large Image Database using Binary Codes

- Map images into binary codes for fast retrieval.



- Small Codes, Torralba, Fergus, Weiss, CVPR 2008
- Spectral Hashing, Y. Weiss, A. Torralba, R. Fergus, NIPS 2008
- Kulis and Darrell, NIPS 2009, Gong and Lazebnik, CVPR 2011
- Norouzi and Fleet, ICML 2011,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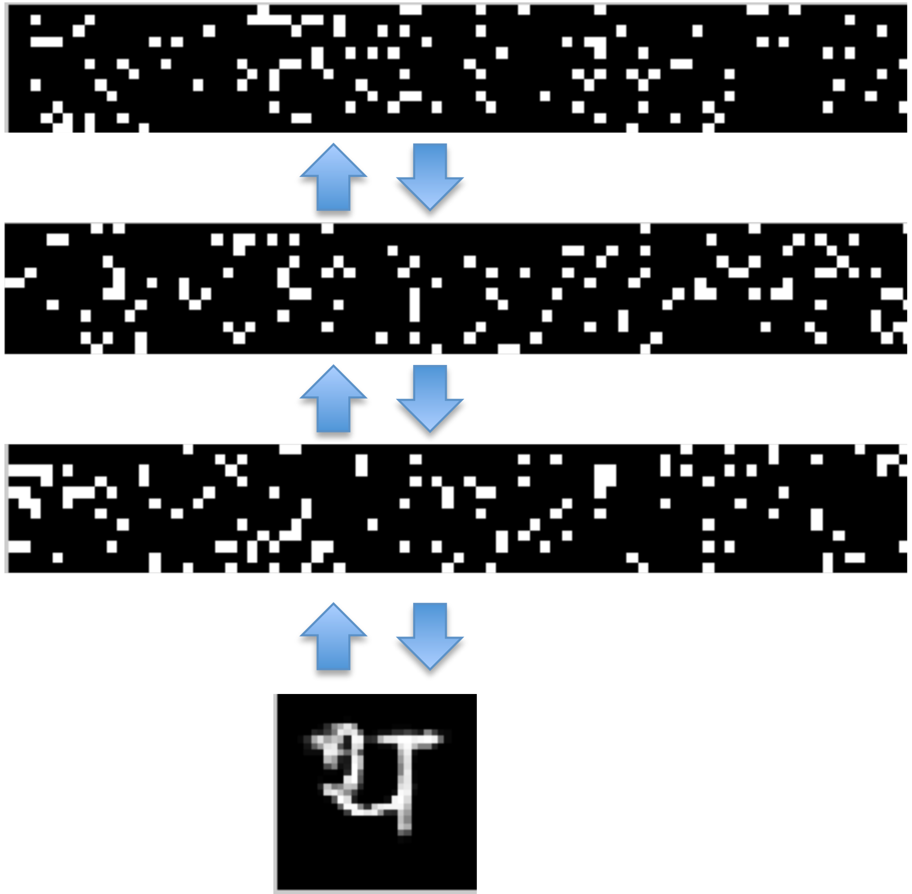
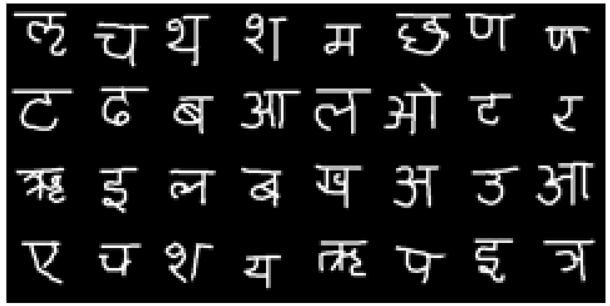
Deep Generative Model

Sanskrit



Model P(image)

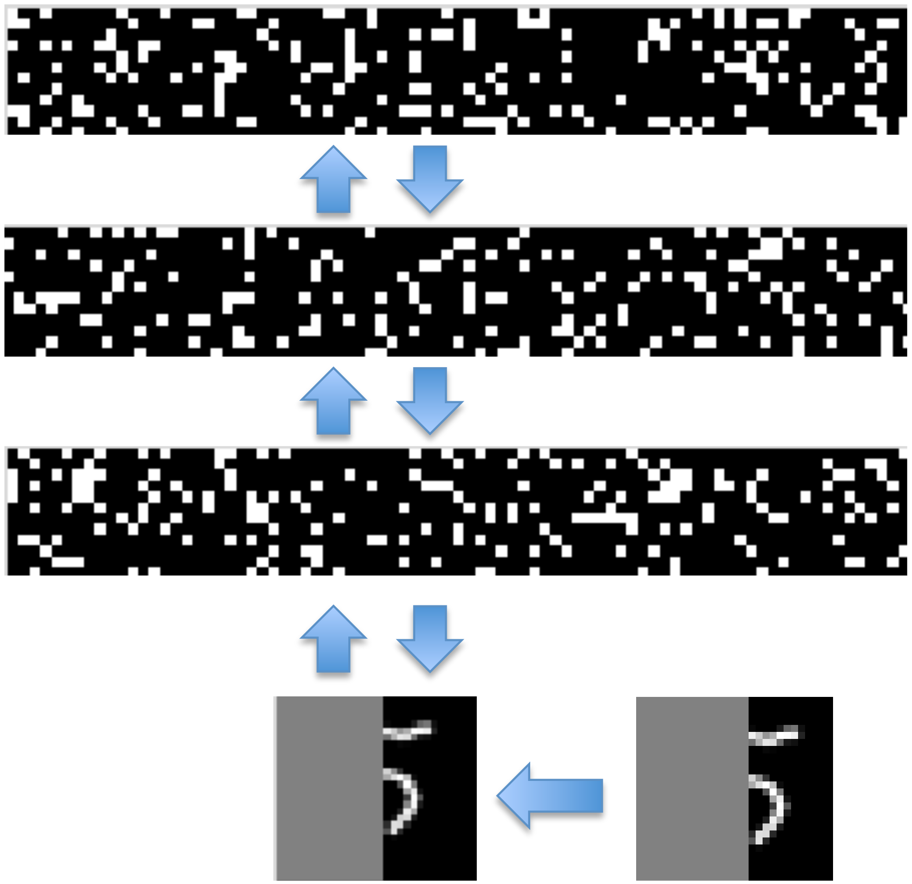


25,000 characters from 50 alphabets around the world.

- 3,000 hidden variables
- 784 observed variables (28 by 28 images)
- About 2 million parameters

Bernoulli Markov Random Field

Deep Generative Model

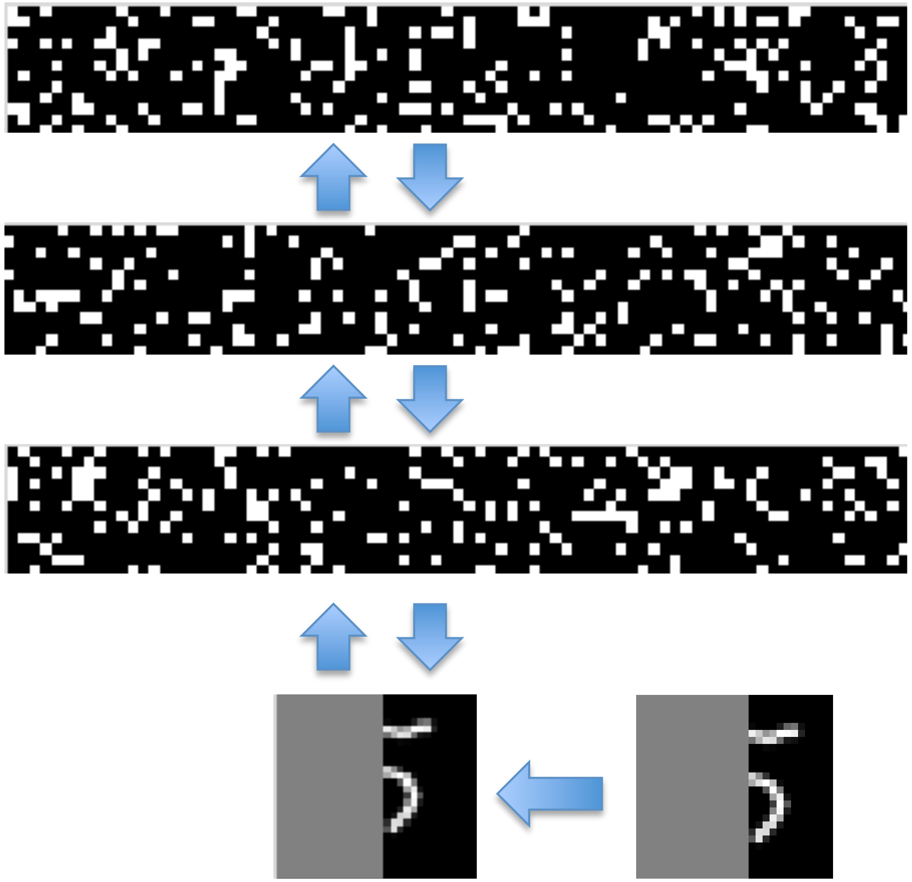


Conditional Simulation

P(image|partial image)

Bernoulli Markov Random Field

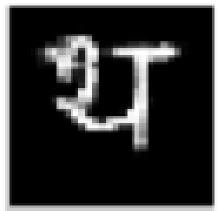
Deep Generative Model



Conditional Simulation

P(image|partial image)

Why so difficult?



$2^{28\times 28}$ possible images!

Bernoulli Markov Random Field

Fully Observed Models

- Explicitly model conditional probabilities:

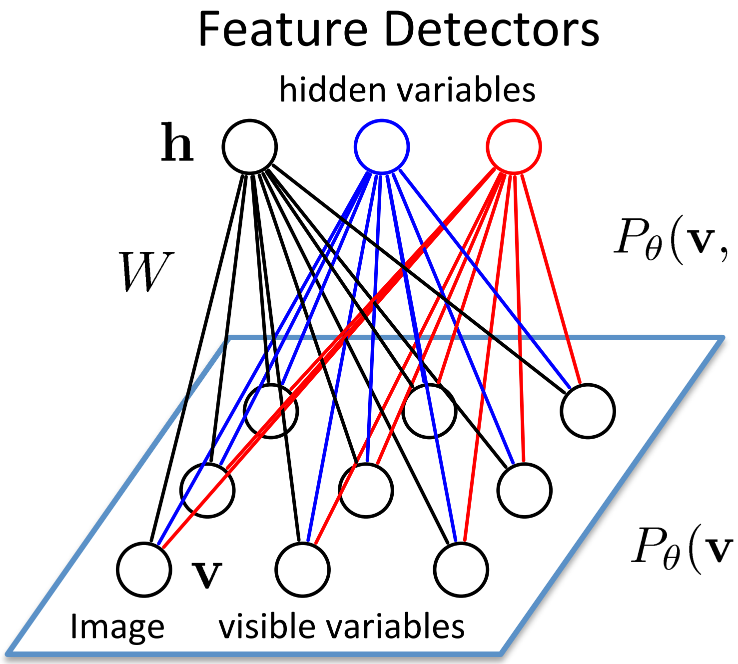
$$p_{\text{model}}(\boldsymbol{x}) = p_{\text{model}}(x_1)\prod_{i=2}^{n} p_{\text{model}}(x_i \mid x_1, \ldots, x_{i-1})$$

Each conditional can be a complicated neural network

- A number of successful models, including
  - NADE, RNADE (Larochelle, et.al. 20011)
  - Pixel CNN (van den Ord et. al. 2016)
  - Pixel RNN (van den Ord et. al. 2016)

Pixel CNN

Restricted Boltzmann Machines



$$P_\theta(\mathbf{v},\mathbf{h}) = \frac{1}{\mathcal{Z}(\theta)} \exp\left(\sum_{i=1}^{D}\sum_{j=1}^{F} \overbrace{W_{ij} v_i h_j}^{\textbf{Pair-wise}} + \overbrace{\sum_{i=1}^{D} v_i b_i}^{\textbf{Unary}} + \overbrace{\sum_{j=1}^{F} h_j a_j} \right)$$

$$\theta = \{W, a, b\}$$

$$P_\theta(\mathbf{v}|\mathbf{h}) = \prod_{i=1}^{D} P_\theta(v_i|\mathbf{h}) = \prod_{i=1}^{D} \frac{1}{1 + \exp(-\sum_{j=1}^{F} W_{ij} v_i h_j - b_i)}$$

RBM is a Markov Random Field with:

- Stochastic binary visible variables $\mathbf{v} \in \{0,1\}^D$.
- Stochastic binary hidden variables $\mathbf{h} \in \{0,1\}^F$.
- Bipartite connections.

Markov random fields, Boltzmann machines, log-linear models.

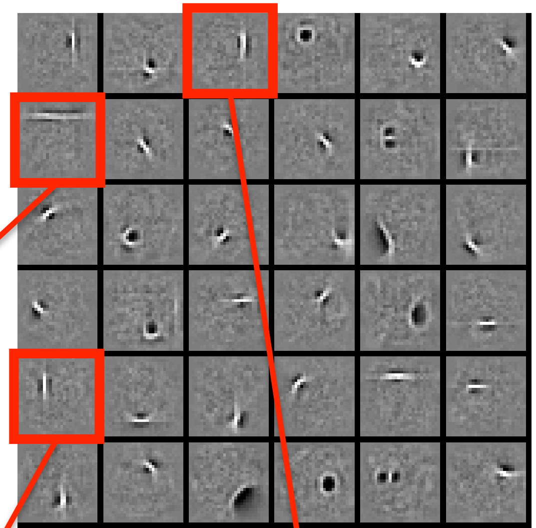
Learning Features

Observed Data

Subset of 25,000 characters

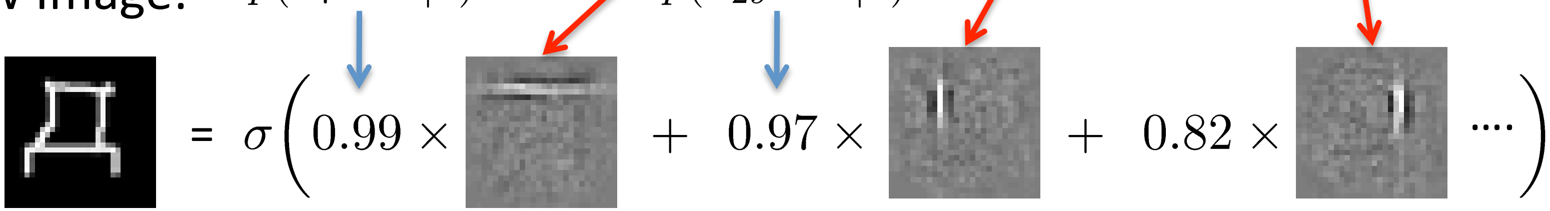
Learned W: "edges"

Subset of 1000 features



New Image:

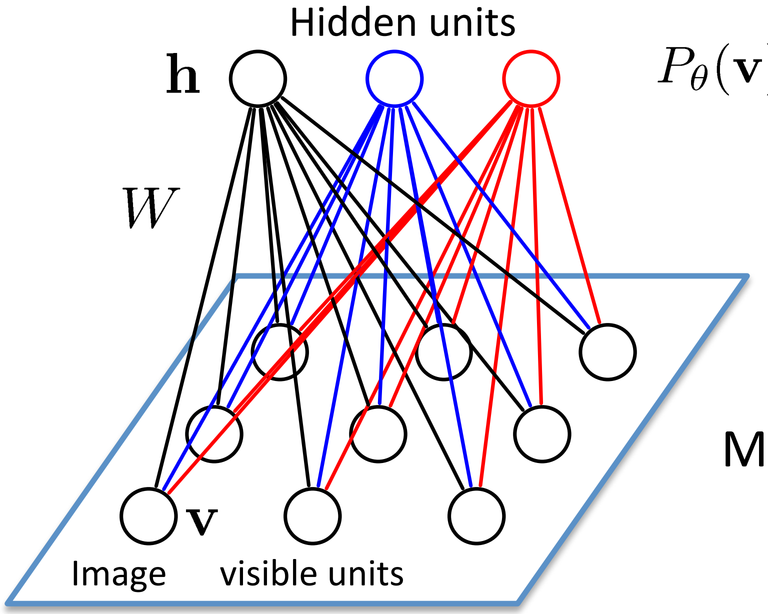
$p(h_7 = 1|v)$ $\quad$ $p(h_{29} = 1|v)$



$\sigma(x) = \frac{1}{1+\exp(-x)}$

Logistic Function: Suitable for modeling binary images

Model Learning



$$P_\theta(\mathbf{v}) = \frac{P^*(\mathbf{v})}{\mathcal{Z}(\theta)} = \frac{1}{\mathcal{Z}(\theta)} \sum_{\mathbf{h}} \exp\left[\mathbf{v}^\top W \mathbf{h} + \mathbf{a}^\top \mathbf{h} + \mathbf{b}^\top \mathbf{v}\right]$$

Given a set of *i.i.d.* training examples $\mathcal{D} = \{\mathbf{v}^{(1)}, \mathbf{v}^{(2)}, ..., \mathbf{v}^{(N)}\}$, we want to learn model parameters $\theta = \{W, a, b\}$.

Maximize log-likelihood objective:

$$L(\theta) = \frac{1}{N} \sum_{n=1}^{N} \log P_\theta(\mathbf{v}^{(n)})$$

Derivative of the log-likelihood:

$$\frac{\partial L(\theta)}{\partial W_{ij}} = \frac{1}{N} \sum_{n=1}^{N} \frac{\partial}{\partial W_{ij}} \log\left(\sum_{\mathbf{h}} \exp\left[\mathbf{v}^{(n)\top} W \mathbf{h} + \mathbf{a}^\top \mathbf{h} + \mathbf{b}^\top \mathbf{v}^{(n)}\right]\right) - \frac{\partial}{\partial W_{ij}} \log \mathcal{Z}(\theta)$$

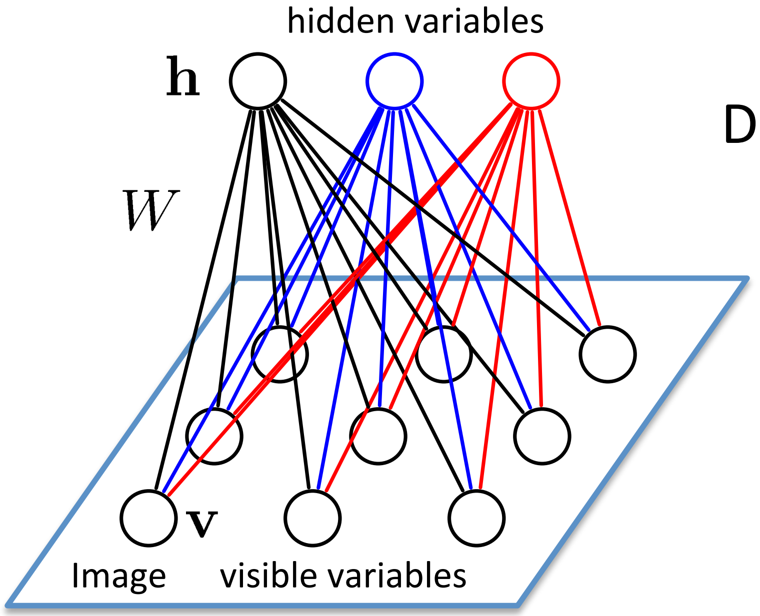
$$= \mathrm{E}_{P_{data}}[v_i h_j] - \underbrace{\mathrm{E}_{P_\theta}[v_i h_j]}$$

Difficult to compute: exponentially many configurations

$$P_{data}(\mathbf{v}, \mathbf{h}; \theta) = P(\mathbf{h}|\mathbf{v}; \theta) P_{data}(\mathbf{v})$$

$$P_{data}(\mathbf{v}) = \frac{1}{N} \sum_{n} \delta(\mathbf{v} - \mathbf{v}^{(n)})$$

Model Learning



Derivative of the log-likelihood:

$$\frac{\partial L(\theta)}{\partial W_{ij}} = \mathbb{E}_{P_{data}}[v_i h_j] - \mathbb{E}_{P_\theta}[v_i h_j]$$

$$\sum_{\mathbf{v},\mathbf{h}} v_i h_j P_\theta(\mathbf{v}, \mathbf{h})$$

Easy to compute exactly

Difficult to compute: exponentially many configurations.

Use MCMC

$$P_{data}(\mathbf{v}, \mathbf{h}; \theta) = P(\mathbf{h}|\mathbf{v}; \theta) P_{data}(\mathbf{v})$$

$$P_{data}(\mathbf{v}) = \frac{1}{N} \sum_n \delta(\mathbf{v} - \mathbf{v}^{(n)})$$

Approximate maximum likelihood learning

Approximate Learning

- An approximation to the gradient of the log-likelihood objective:

$$\frac{\partial L(\theta)}{\partial W_{ij}} = \mathbb{E}_{P_{data}}[v_i h_j] - \mathbb{E}_{P_\theta}[v_i h_j]$$

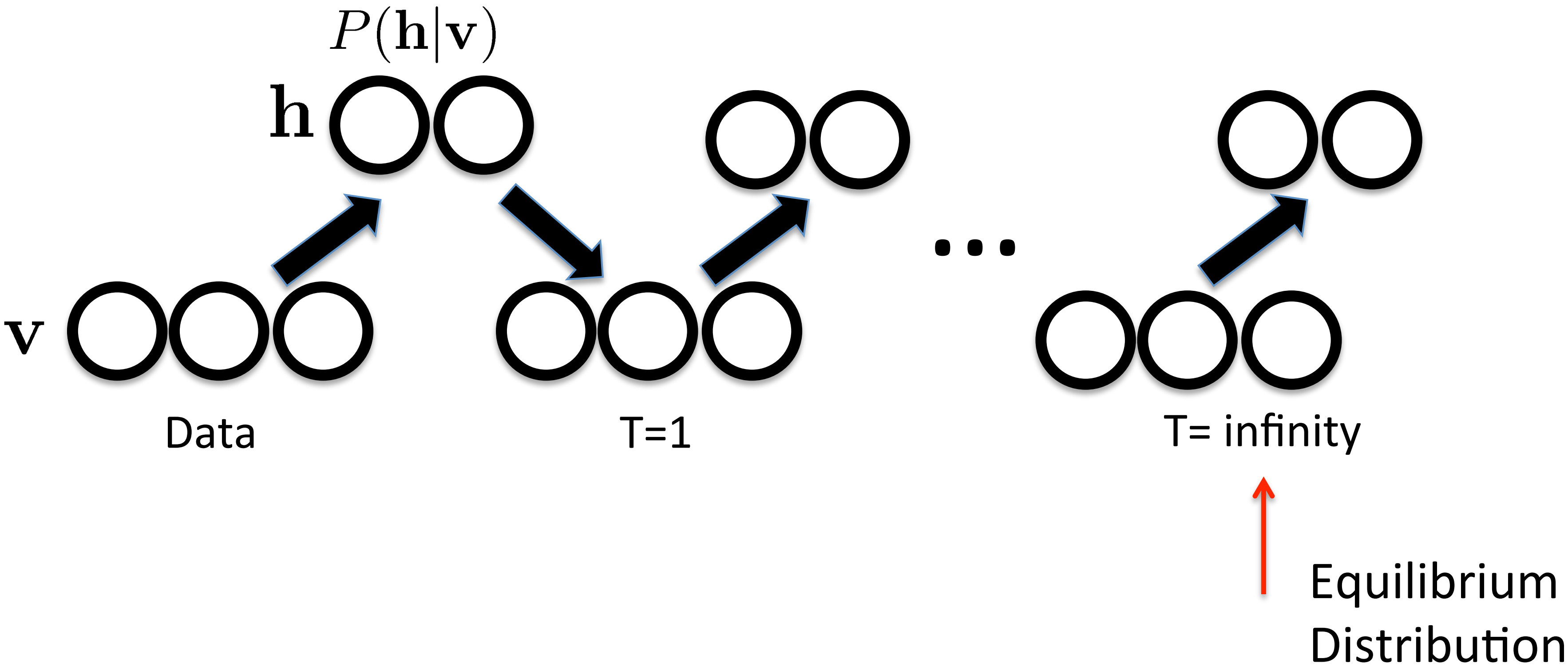
$$\sum_{\mathbf{v},\mathbf{h}} v_i h_j P_\theta(\mathbf{v}, \mathbf{h})$$

- Replace the average over all possible input configurations by samples.
- Run MCMC chain (Gibbs sampling) starting from the observed examples.

- Initialize $v^0 = v$
- Sample h^0 from $P(h \mid v^0)$
- For t=1:T
  - Sample v^t from $P(v \mid h^{t-1})$
  - Sample h^t from $P(h \mid v^t)$

Approximate ML Learning for RBMs

Run Markov chain (alternating Gibbs Sampling):

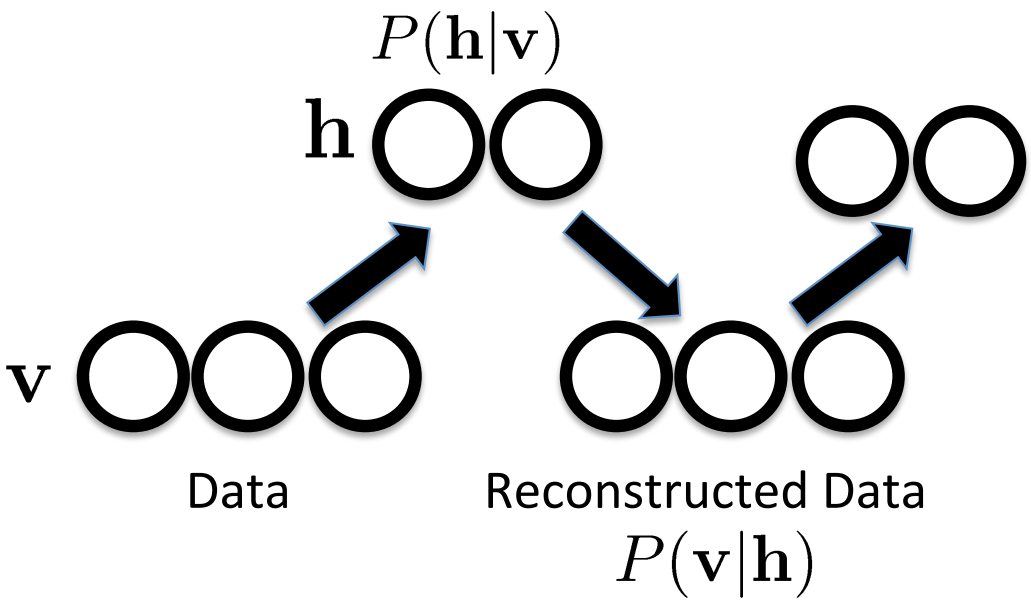


$$P(\mathbf{h}|\mathbf{v}) = \prod_j P(h_j|\mathbf{v}) \quad P(h_j = 1|\mathbf{v}) = \frac{1}{1 + \exp(-\sum_i W_{ij} v_i - a_j)}$$

$$P(\mathbf{v}|\mathbf{h}) = \prod_i P(v_i|\mathbf{h}) \quad P(v_i = 1|\mathbf{h}) = \frac{1}{1 + \exp(-\sum_j W_{ij} h_j - b_i)}$$

Contrastive Divergence

A quick way to learn RBM:



- Start with a training vector on the visible units.
- Update all the hidden units in parallel.
- Update the all the visible units in parallel to get a "reconstruction".
- Update the hidden units again.

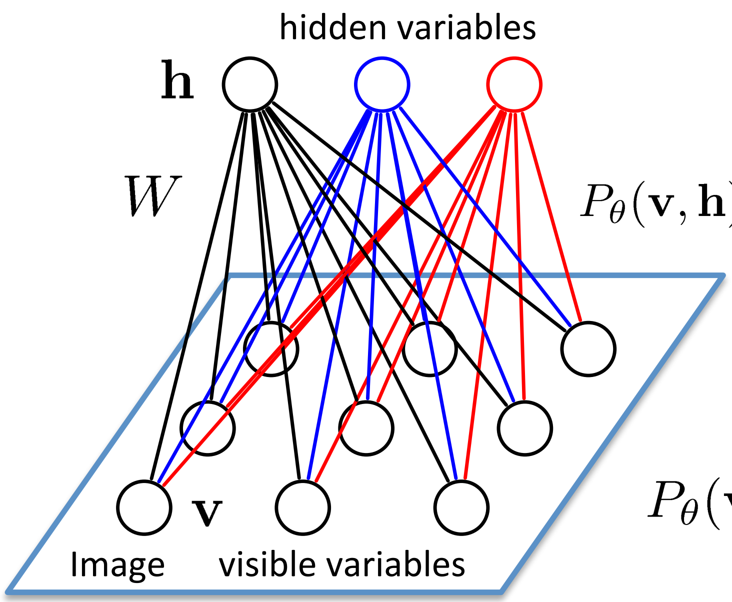
Update model parameters:

$$\Delta W_{ij} = \mathrm{E}_{P_{data}}[v_i h_j] - \mathrm{E}_{P_1}[v_i h_j]$$

Implementation: ~10 lines of Matlab code.

(Hinton, Neural Computation 2002)

RBMs for Real-valued Data



$$P_\theta(\mathbf{v}, \mathbf{h}) = \frac{1}{\mathcal{Z}(\theta)} \exp\left(\sum_{i=1}^{D} \sum_{j=1}^{F} \overbrace{W_{ij} h_j \frac{v_i}{\sigma_i}}^{\textbf{Pair-wise}} + \sum_{i=1}^{D} \overbrace{\frac{(v_i - b_i)^2}{2\sigma_i^2}}^{\textbf{Unary}} + \sum_{j=1}^{F} \overbrace{a_j h_j} \right)$$

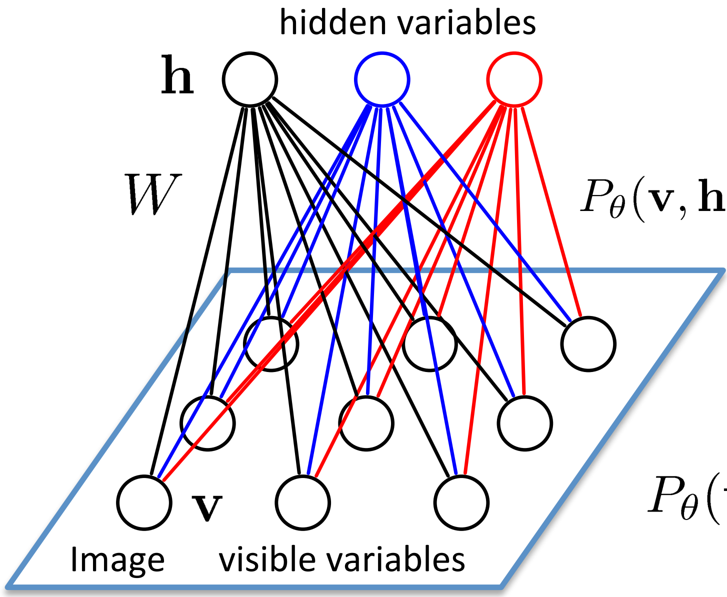
$$\theta = \{W, a, b\}$$

$$P_\theta(\mathbf{v}|\mathbf{h}) = \prod_{i=1}^{D} P_\theta(v_i|\mathbf{h}) = \prod_{i=1}^{D} \mathcal{N}\left(b_i + \sum_{j=1}^{F} W_{ij} h_j, \sigma_i^2 \right)$$

Gaussian-Bernoulli RBM:

- Stochastic real-valued visible variables $\mathbf{v} \in \mathbb{R}^D$.
- Stochastic binary hidden variables $\mathbf{h} \in \{0, 1\}^F$.
- Bipartite connections.

RBMs for Real-valued Data

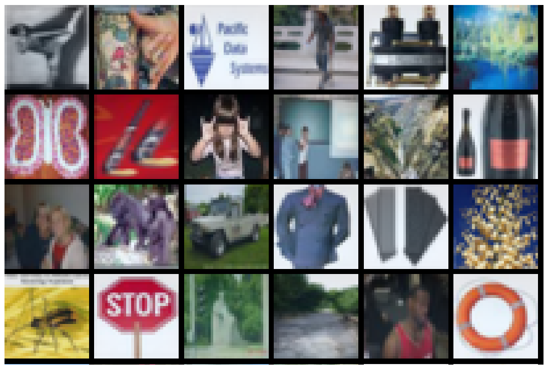


$$P_\theta(\mathbf{v},\mathbf{h}) = \frac{1}{\mathcal{Z}(\theta)} \exp\left(\sum_{i=1}^{D}\sum_{j=1}^{F} \overbrace{W_{ij} h_j \frac{v_i}{\sigma_i}}^{\textbf{Pair-wise}} + \overbrace{\sum_{i=1}^{D} \frac{(v_i - b_i)^2}{2\sigma_i^2} + \sum_{j=1}^{F} a_j h_j}^{\textbf{Unary}} \right)$$

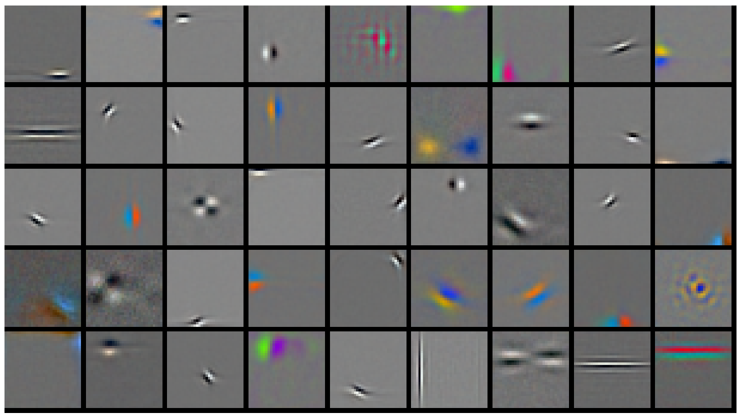
$$\theta = \{W, a, b\}$$

$$P_\theta(\mathbf{v}|\mathbf{h}) = \prod_{i=1}^{D} P_\theta(v_i|\mathbf{h}) = \prod_{i=1}^{D} \mathcal{N}\left(b_i + \sum_{j=1}^{F} W_{ij} h_j, \sigma_i^2 \right)$$

4 million **unlabelled** images

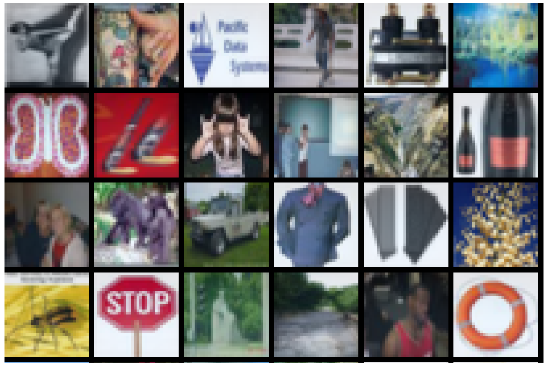


Learned features (out of 10,000)

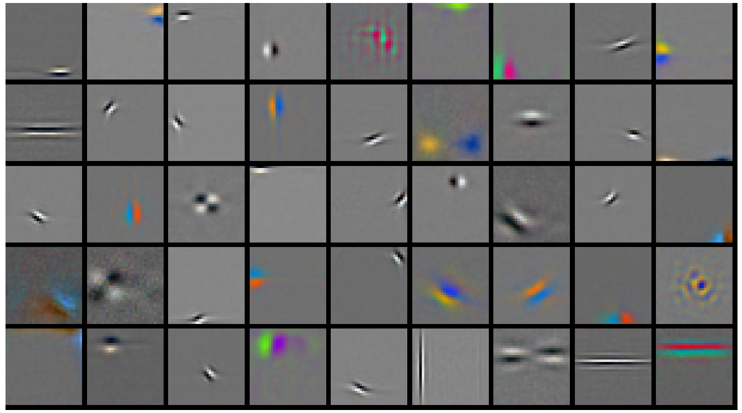


RBMs for Real-valued Data

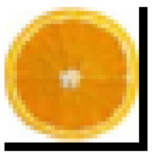
4 million **unlabelled** images

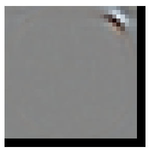
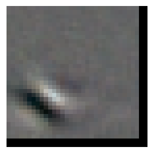
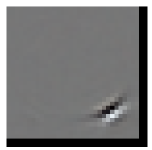


Learned features (out of 10,000)



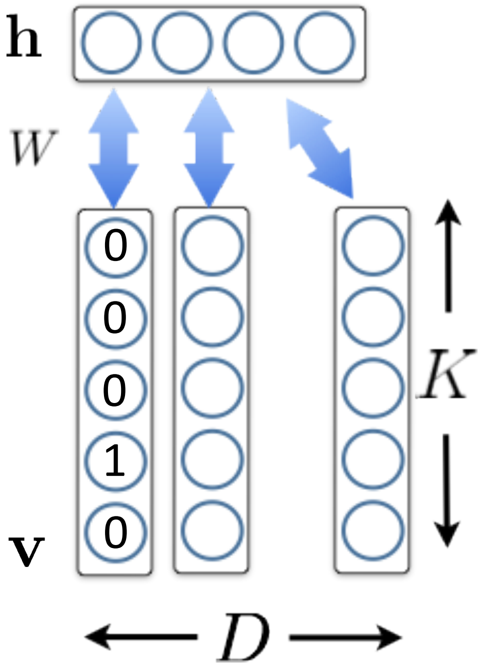
$p(h_7 = 1|v)$ $p(h_{29} = 1|v)$



$= 0.9\, *$  $+\ 0.8\, *$  $+\, 0.6\, *$  $\ldots$

New Image

RBMs for Word Counts



$$P_\theta(\mathbf{v},\mathbf{h}) = \frac{1}{\mathcal{Z}(\theta)} \exp\left(\overbrace{\sum_{i=1}^{D}\sum_{k=1}^{K}\sum_{j=1}^{F} W_{ij}^{k} v_i^k h_j}^{\textbf{Pair-wise}} + \overbrace{\sum_{i=1}^{D}\sum_{k=1}^{K} v_i^k b_i^k + \sum_{j=1}^{F} h_j a_j}^{\textbf{Unary}} \right)$$

$$\theta = \{W, a, b\}$$

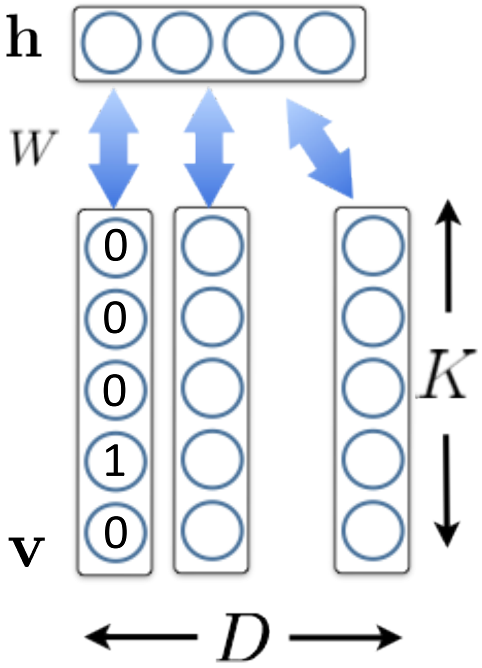
$$P_\theta(v_i^k = 1|\mathbf{h}) = \frac{\exp\left(b_i^k + \sum_{j=1}^{F} h_j W_{ij}^k\right)}{\sum_{q=1}^{K} \exp\left(b_i^q + \sum_{j=1}^{F} h_j W_{ij}^q\right)}$$

Replicated Softmax Model: undirected topic model:

- Stochastic 1-of-K visible variables.
- Stochastic binary hidden variables $\mathbf{h} \in \{0,1\}^F$.
- Bipartite connections.

(Salakhutdinov & Hinton, NIPS 2010, Srivastava & Salakhutdinov, NIPS 2012)

RBMs for Word Counts



$$P_\theta(\mathbf{v},\mathbf{h}) = \frac{1}{\mathcal{Z}(\theta)} \exp\left(\overbrace{\sum_{i=1}^{D}\sum_{k=1}^{K}\sum_{j=1}^{F} W_{ij}^{k} v_i^k h_j}^{\text{Pair-wise}} + \overbrace{\sum_{i=1}^{D}\sum_{k=1}^{K} v_i^k b_i^k + \sum_{j=1}^{F} h_j a_j}^{\text{Unary}} \right)$$

$$\theta = \{W, a, b\}$$

$$P_\theta(v_i^k = 1|\mathbf{h}) = \frac{\exp\left(b_i^k + \sum_{j=1}^{F} h_j W_{ij}^k\right)}{\sum_{q=1}^{K} \exp\left(b_i^q + \sum_{j=1}^{F} h_j W_{ij}^q\right)}$$

REUTERS

Associated Press

WIKIPEDIA The Free Encyclopedia

Reuters dataset:
804,414 **unlabeled**
newswire stories
Bag-of-Words

Learned features: ``topics''

russian	clinton	computer	trade	stock
russia	house	system	country	wall
moscow	president	product	import	street
yeltsin	bill	software	world	point
soviet	congress	develop	economy	dow

RBMs for Word Counts

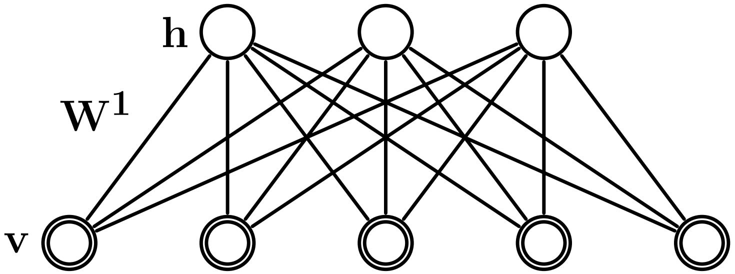
One-step reconstruction from the Replicated Softmax model.

Input	Reconstruction
chocolate, cake	cake, chocolate, sweets, dessert, cupcake, food, sugar, cream, birthday
nyc	nyc, newyork, brooklyn, queens, gothamist, manhattan, subway, streetart
dog	dog, puppy, perro, dogs, pet, filmshots, tongue, pets, nose, animal
flower, high, 花	flower, 花, high, japan, sakura, 日本, blossom, tokyo, lily, cherry
girl, rain, station, norway	norway, station, rain, girl, oslo, train, umbrella, wet, railway, weather
fun, life, children	children, fun, life, kids, child, playing, boys, kid, play, love
forest, blur	forest, blur, woods, motion, trees, movement, path, trail, green, focus
españa, agua, granada	españa, agua, spain, granada, water, andalucía, naturaleza, galicia, nieve

Collaborative Filtering

$$P_\theta(\mathbf{v},\mathbf{h}) = \frac{1}{\mathcal{Z}(\theta)} \exp\left(\sum_{ijk} W_{ij}^k v_i^k h_j + \sum_{ik} b_i^k v_i^k + \sum_j a_j h_j \right)$$

Binary hidden: user preferences



Multinomial visible: user ratings

Netflix dataset:
480,189 users
17,770 movies
Over 100 million ratings

Learned features: ``genre''

Fahrenheit 9/11
Bowling for Columbine
The People vs. Larry Flynt
Canadian Bacon
La Dolce Vita

Independence Day
The Day After Tomorrow
Con Air
Men in Black II
Men in Black

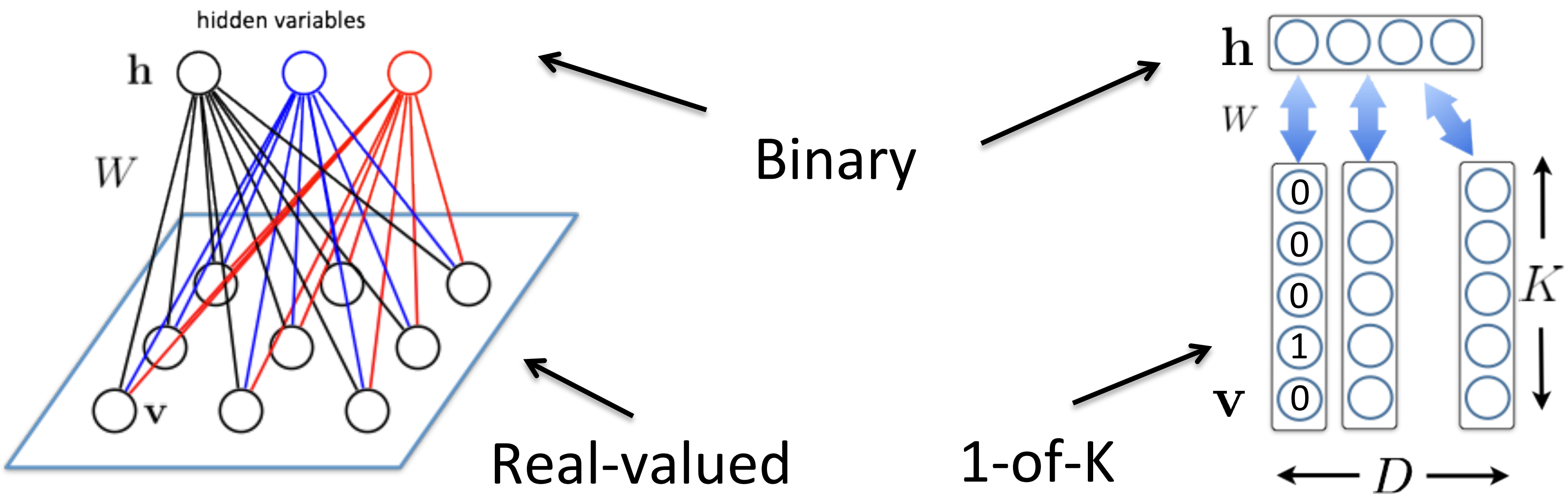
Friday the 13th
The Texas Chainsaw Massacre
Children of the Corn
Child's Play
The Return of Michael Myers

Scary Movie
Naked Gun
Hot Shots!
American Pie
Police Academy

(Salakhutdinov, Mnih, Hinton, ICML 2007)

Different Data Modalities

- Binary/Gaussian/Softmax RBMs: All have binary hidden variables but use them to model different kinds of data.



- It is easy to infer the states of the hidden variables:

$$P_\theta(\mathbf{h}|\mathbf{v}) = \prod_{j=1}^{F} P_\theta(h_j|\mathbf{v}) = \prod_{j=1}^{F} \frac{1}{1+\exp(-a_j - \sum_{i=1}^{D} W_{ij} v_i)}$$

Product of Experts

The joint distribution is given by:

$$P_\theta(\mathbf{v},\mathbf{h}) = \frac{1}{\mathcal{Z}(\theta)} \exp\Big(\sum_{ij} W_{ij} v_i h_j + \sum_i b_i v_i + \sum_j a_j h_j\Big)$$

Marginalizing over hidden variables:

$$P_\theta(\mathbf{v}) = \sum_{\mathbf{h}} P_\theta(\mathbf{v},\mathbf{h}) = \frac{1}{\mathcal{Z}(\theta)} \prod_i \exp(b_i v_i) \prod_j \overbrace{\Big(1 + \exp(a_j + \sum_i W_{ij} v_i)\Big)}^{\textbf{Product of Experts}}$$

government	clinton	bribery	mafia	stock	…
authority	house	corruption	business	wall	
power	president	dishonesty	gang	street	
empire	bill	corrupt	mob	point	
federation	congress	fraud	insider	dow	

Silvio Berlusconi

Topics “government”, ”corruption” and ”mafia” can combine to give very high probability to a word “Silvio Berlusconi”.

Product of Experts

The joint distribution is given by:

$$P_\theta(\mathbf{v},\mathbf{h}) = \frac{1}{\mathcal{Z}(\theta)} \exp\Big(\sum_{ij} W_{ij} v_i h_j + \sum_i b_i v_i + \sum_j a_j h_j\Big)$$

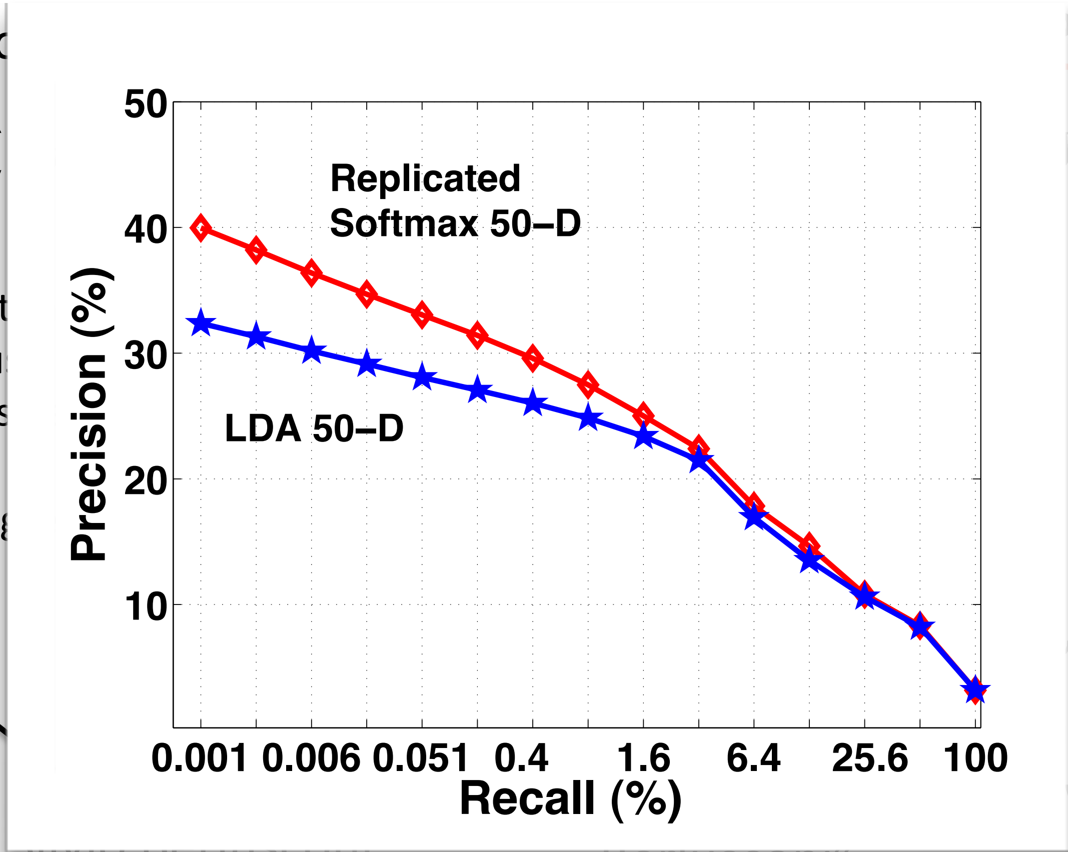
Marginalizing c

duct of Experts

$$P_\theta(\mathbf{v}) = \sum_{\mathbf{h}} \quad \ldots \quad W_{ij} v_i)\Big)$$

government
authority
power
empire
federation

clint
hou
pres
bill
con



, "corruption"
bine to give very
word "Silvio

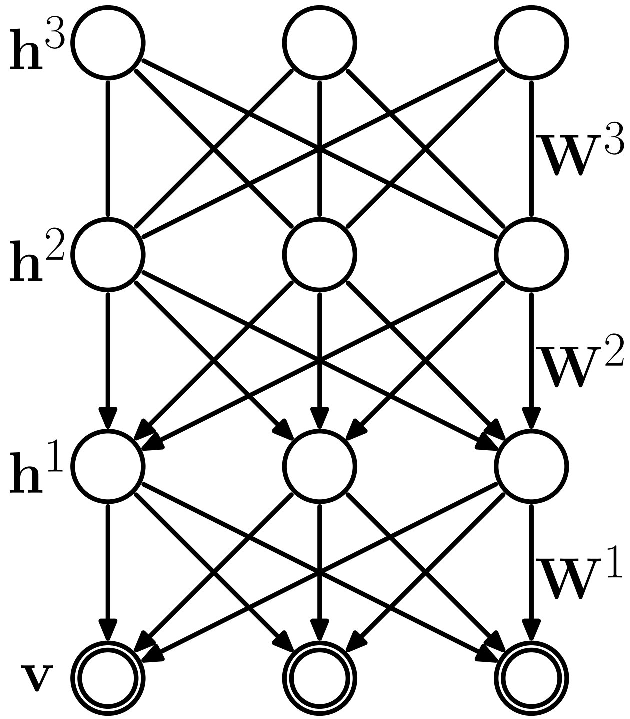
Silvio Berlusconi

Berlusconi".

Talk Roadmap

- Basic Building Blocks (non-probabilistic models):
  - Sparse Coding
  - Autoencoders
- Deep Generative Models
  - Restricted Boltzmann Machines
  - Deep Boltzmann Machines
  - Helmholtz Machines / Variational Autoencoders
- Generative Adversarial Networks

Deep Belief Network

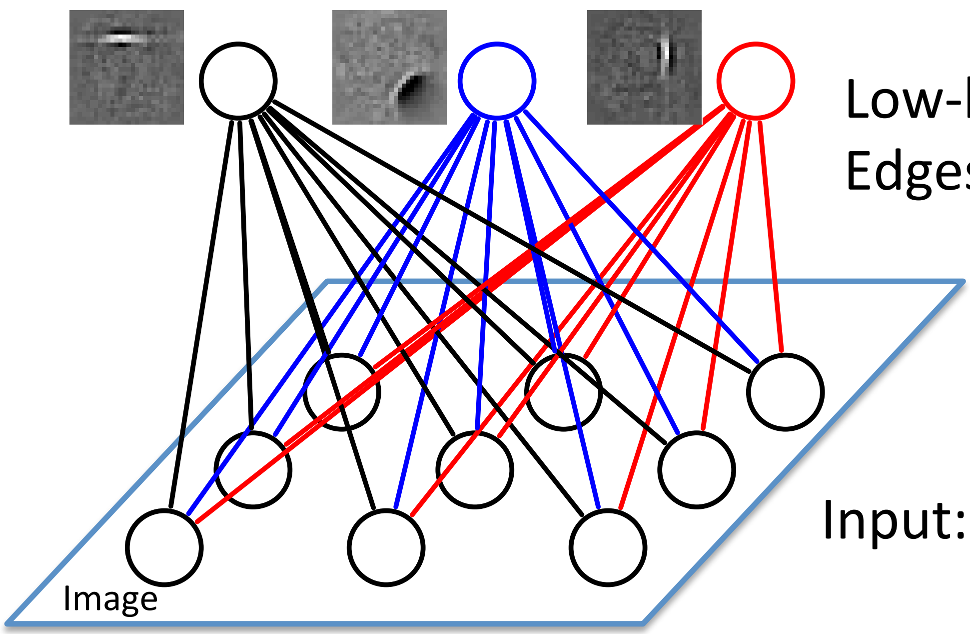


- Probabilistic Generative model.
- Contains multiple layers of nonlinear representation.
- Fast, greedy layer-wise pretraining algorithm.
- Inferring the states of the latent variables in highest layers is easy.

- Inferring the states of the latent variables in highest layers is easy.

(Hinton et al. Neural Computation 2006)

Deep Belief Network



Low-level features:
Edges

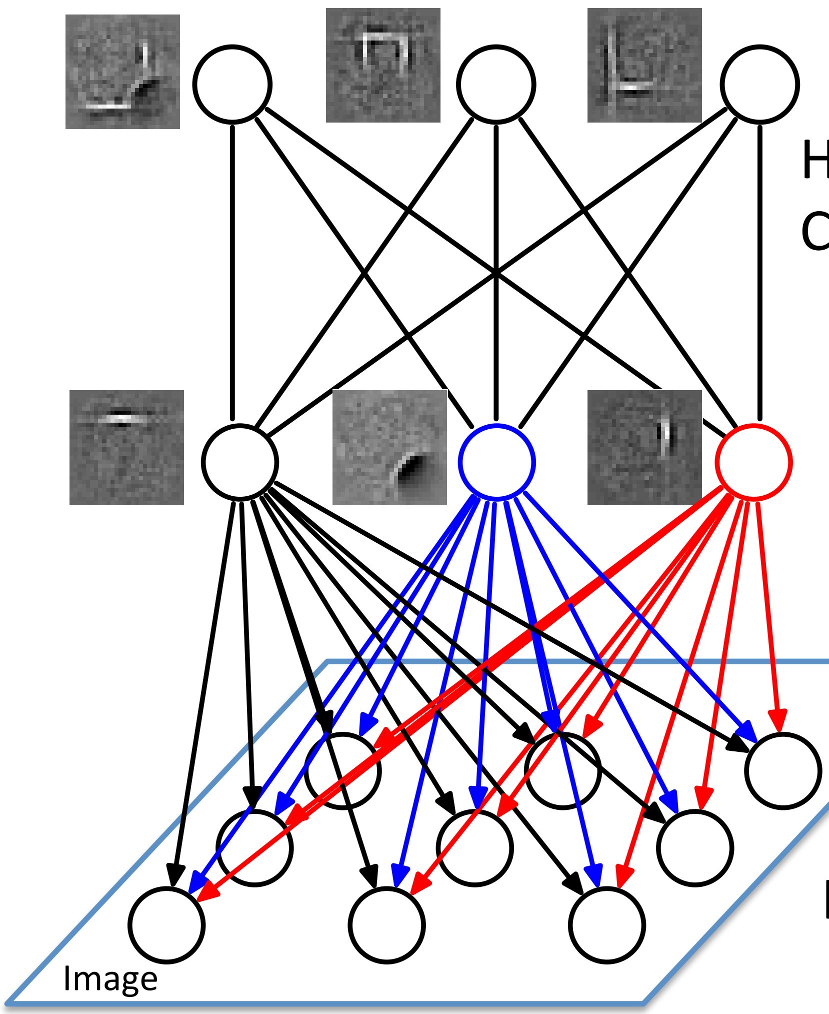
Built from **unlabeled** inputs.

Input: Pixels

(Hinton et al. Neural Computation 2006)

Deep Belief Network

Internal representations capture higher-order statistical structure



Higher-level features:
Combination of edges

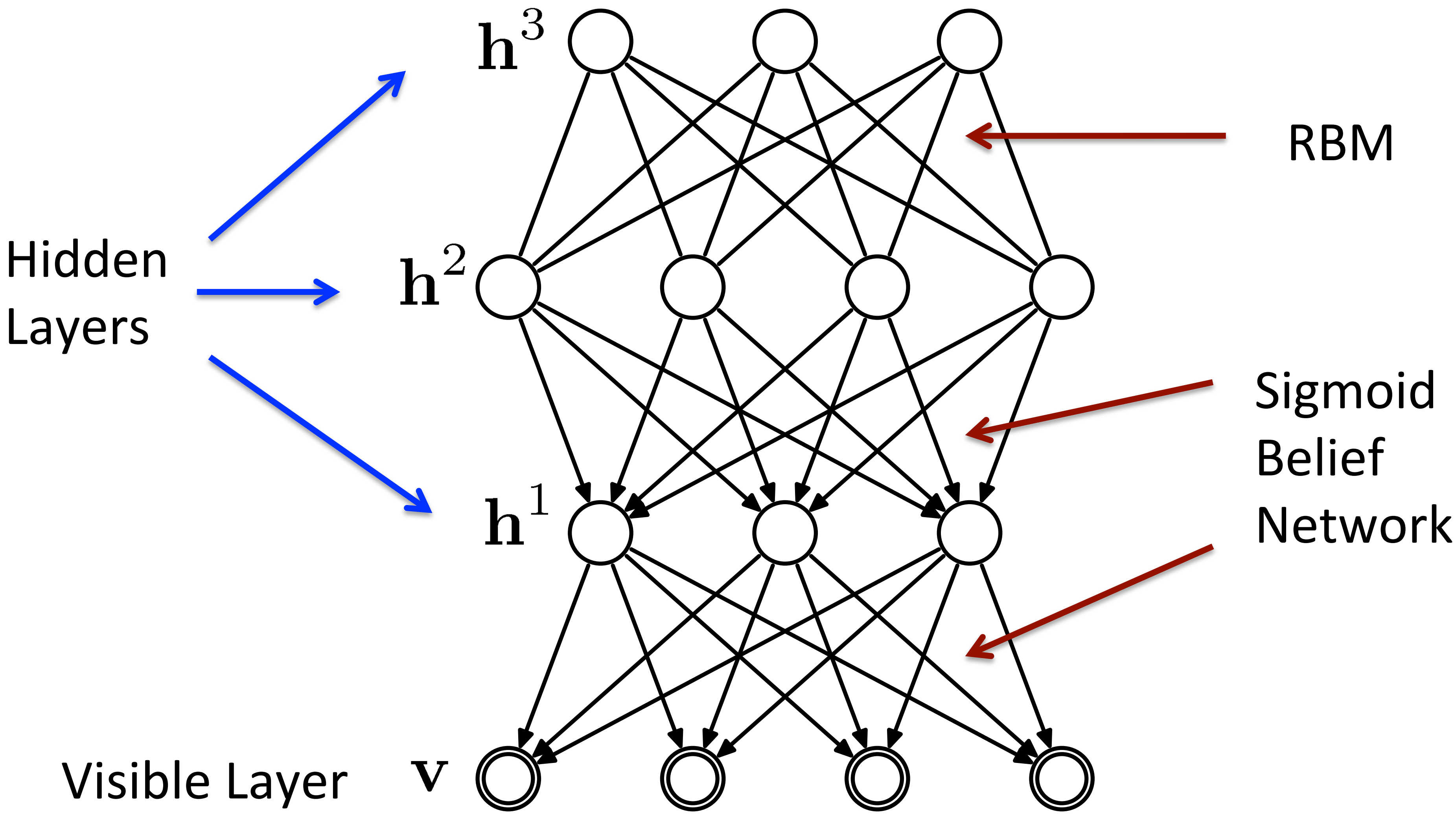
Low-level features:
Edges

Built from **unlabeled** inputs.

Input: Pixels

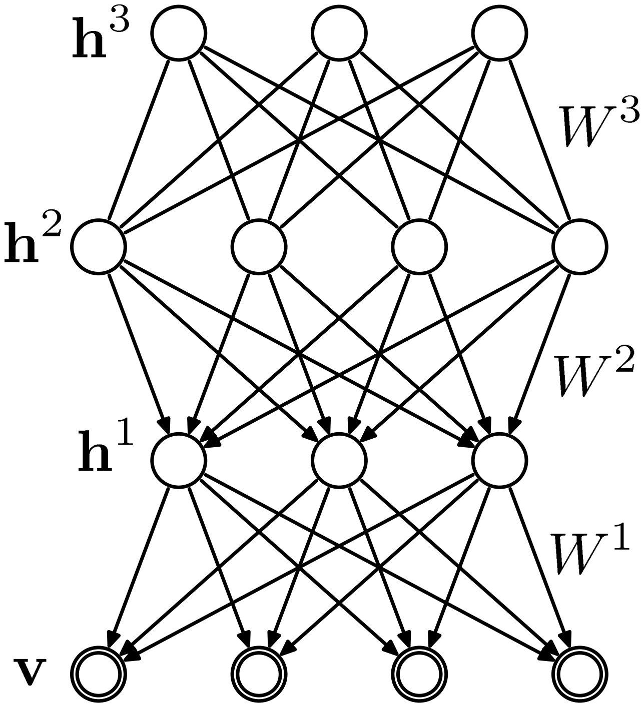
(Hinton et al. Neural Computation 2006)

Deep Belief Network



Deep Belief Network

Deep Belief Network



RBM

Sigmoid Belief Network

The joint probability distribution factorizes:

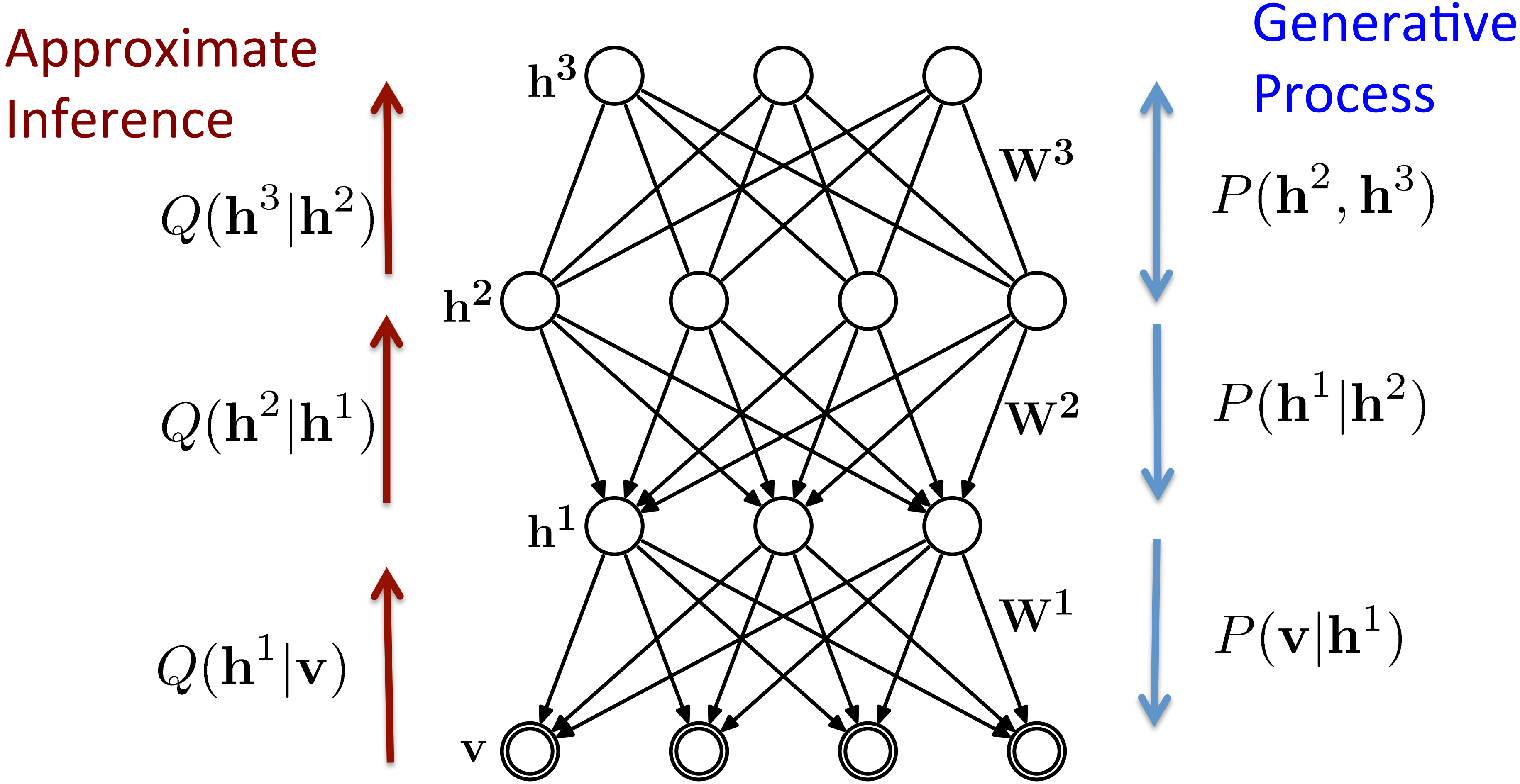
$$P(\mathbf{v}, \mathbf{h}^1, \mathbf{h}^2, \mathbf{h}^3) = \underbrace{P(\mathbf{v}|\mathbf{h}^1) P(\mathbf{h}^1|\mathbf{h}^2)}_{\text{Sigmoid Belief Network}} \underbrace{P(\mathbf{h}^2, \mathbf{h}^3)}_{\text{RBM}}$$

$$P(\mathbf{h}^2, \mathbf{h}^3) = \frac{1}{\mathcal{Z}(W^3)} \exp\left[\mathbf{h}^{2\top} W^3 \mathbf{h}^3\right]$$

$$P(\mathbf{h}^1|\mathbf{h}^2) = \prod_j P(h_j^1|\mathbf{h}^2) \qquad P(h_j^1 = 1|\mathbf{h}^2) = \frac{1}{1 + \exp\left(-\sum_k W_{jk}^2 h_k^2\right)}$$

$$P(\mathbf{v}|\mathbf{h}^1) = \prod_i P(v_i|\mathbf{h}^1) \qquad P(v_i = 1|\mathbf{h}^1) = \frac{1}{1 + \exp\left(-\sum_j W_{ij}^1 h_j^1\right)}$$

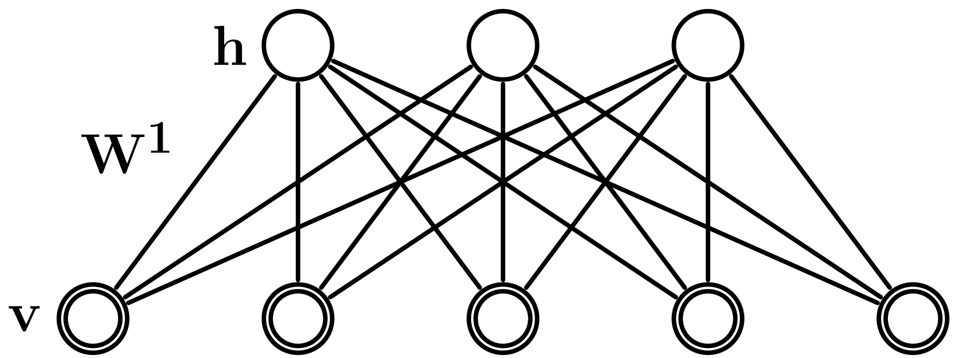
Deep Belief Network



$$Q(\mathbf{h}^t|\mathbf{h}^{t-1}) = \prod_j \sigma\left(\sum_i W^t h_i^{t-1}\right) \qquad P(\mathbf{h}^{t-1}|\mathbf{h}^t) = \prod_j \sigma\left(\sum_i W^t h_i^t\right)$$

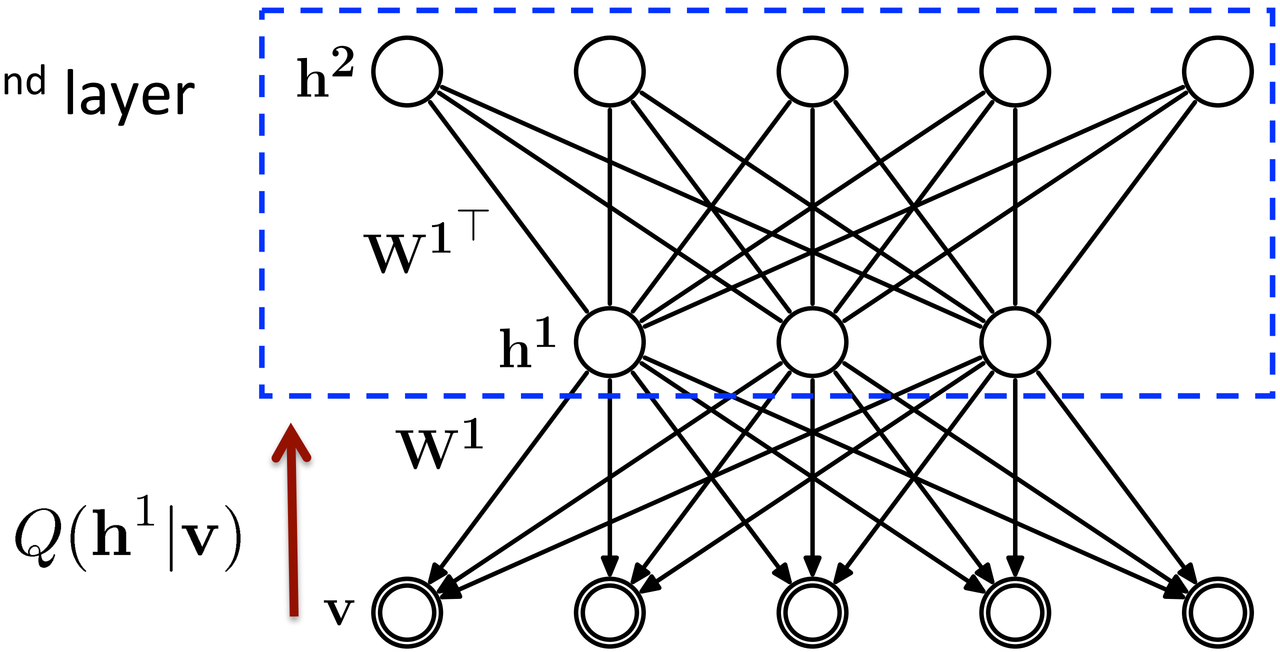
DBN Layer-wise Training

- Learn an RBM with an input layer v and a hidden layer h.



DBN Layer-wise Training

- Learn an RBM with an input layer v and a hidden layer h.
- Treat inferred values $Q(\mathbf{h}^1|\mathbf{v}) = P(\mathbf{h}^1|\mathbf{v})$ as the data for training 2nd-layer RBM.
- Learn and freeze 2nd layer RBM.



DBN Layer-wise Training

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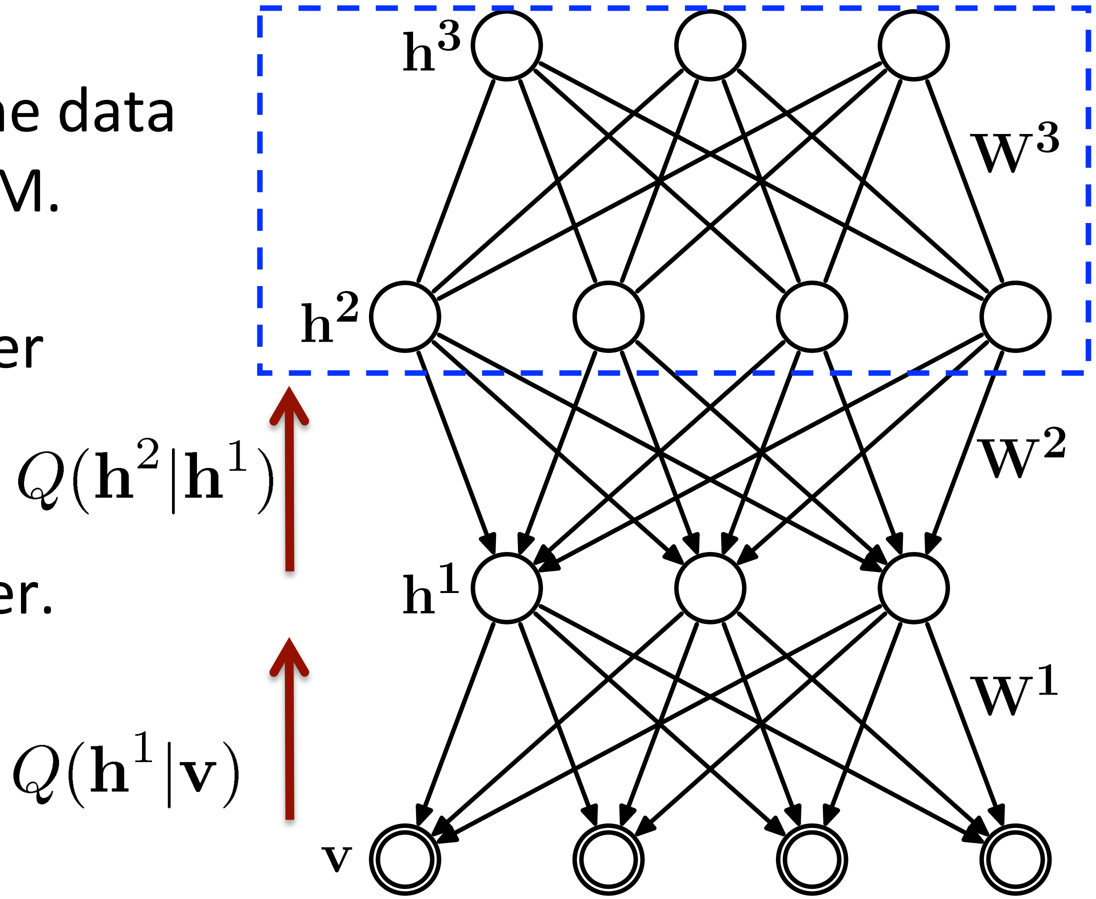
- Learn and freeze 2nd layer RBM.
- Proceed to the next layer.

Unsupervised Feature Learning.



DBN Layer-wise Training

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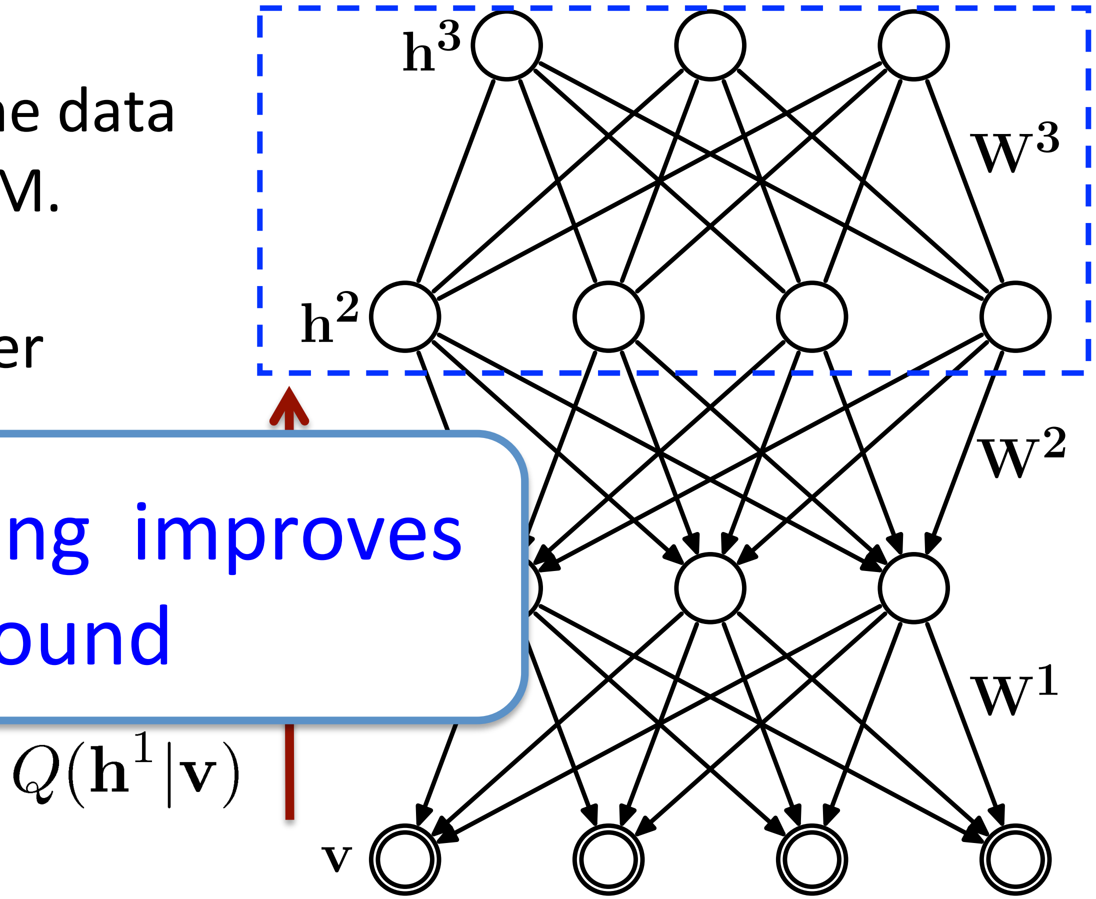
Unsupervised Feature Learning.

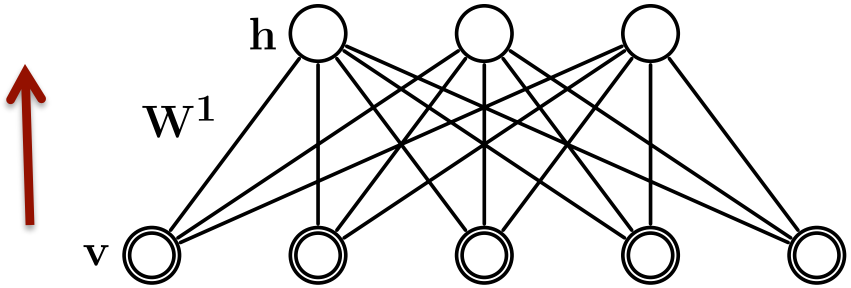


Layerwise pretraining improves variational lower bound

Why this Pre-training Works?

- Greedy training improves variational lower bound!

$Q(\mathbf{h}^1|\mathbf{v})$



- For any approximating distribution $Q(\mathbf{h}^1|\mathbf{v})$

$$\log P_\theta(\mathbf{v}) = \sum_{\mathbf{h}^1} P_\theta(\mathbf{v}, \mathbf{h}^1)$$

$$\geq \sum_{\mathbf{h}^1} Q(\mathbf{h}^1|\mathbf{v}) \Big[\log P(\mathbf{h}^1) + \log P(\mathbf{v}|\mathbf{h}^1) \Big] + \mathcal{H}(Q(\mathbf{h}^1|\mathbf{v}))$$

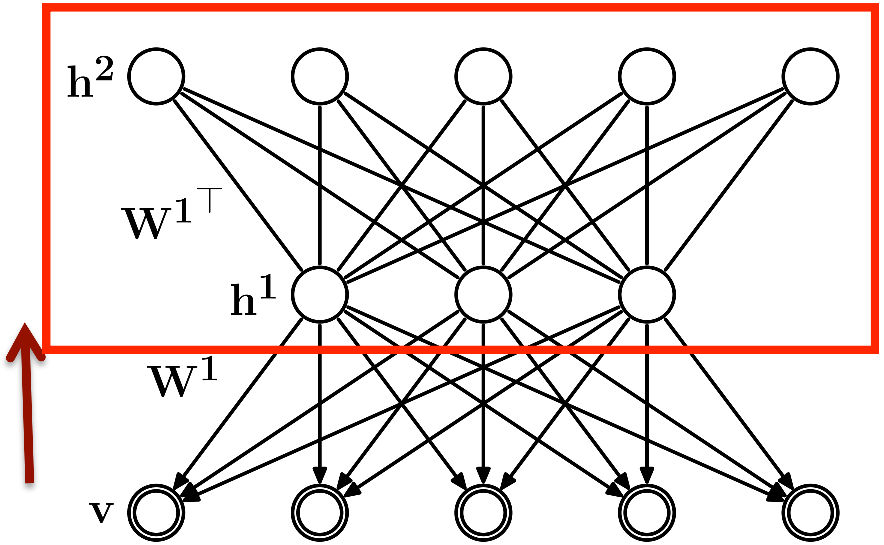
Why this Pre-training Works?

- Greedy training improves variational lower bound.

- RBM and 2-layer DBN are equivalent when $W^2 = {W^1}^\top$.

- The lower bound is tight and the log-likelihood improves by greedy training.

$Q(\mathbf{h}^1|\mathbf{v})$



- For any approximating distribution $Q(\mathbf{h}^1|\mathbf{v})$

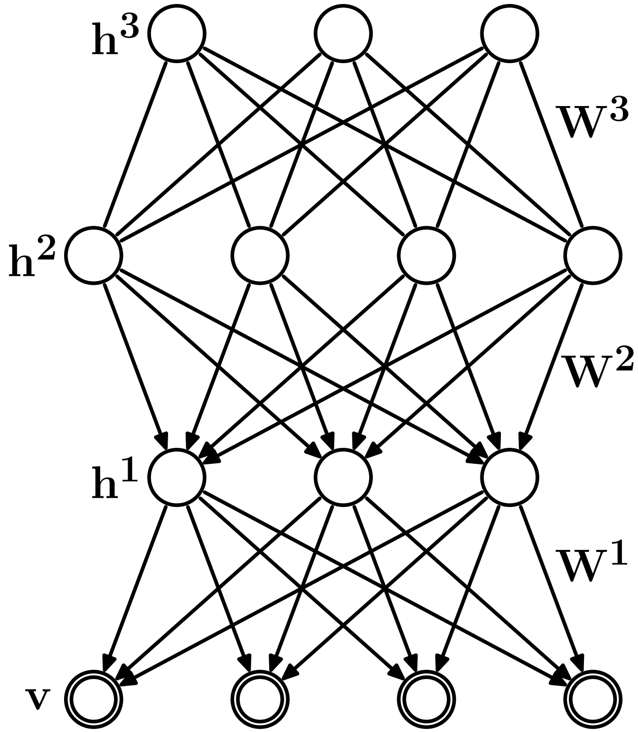
$$\log P_\theta(\mathbf{v}) = \sum_{\mathbf{h}^1} P_\theta(\mathbf{v}, \mathbf{h}^1)$$

Train 2nd-layer RBM

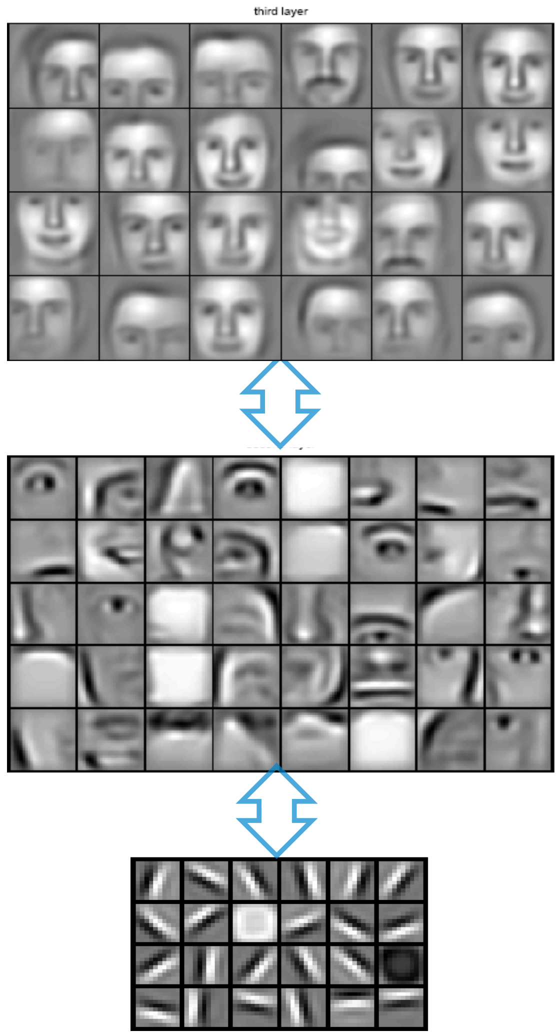
$$\geq \sum_{\mathbf{h}^1} Q(\mathbf{h}^1|\mathbf{v}) \Big[\log P(\mathbf{h}^1) + \log P(\mathbf{v}|\mathbf{h}^1) \Big] + \mathcal{H}(Q(\mathbf{h}^1|\mathbf{v}))$$

Learning Part-based Representation

Convolutional DBN



Faces



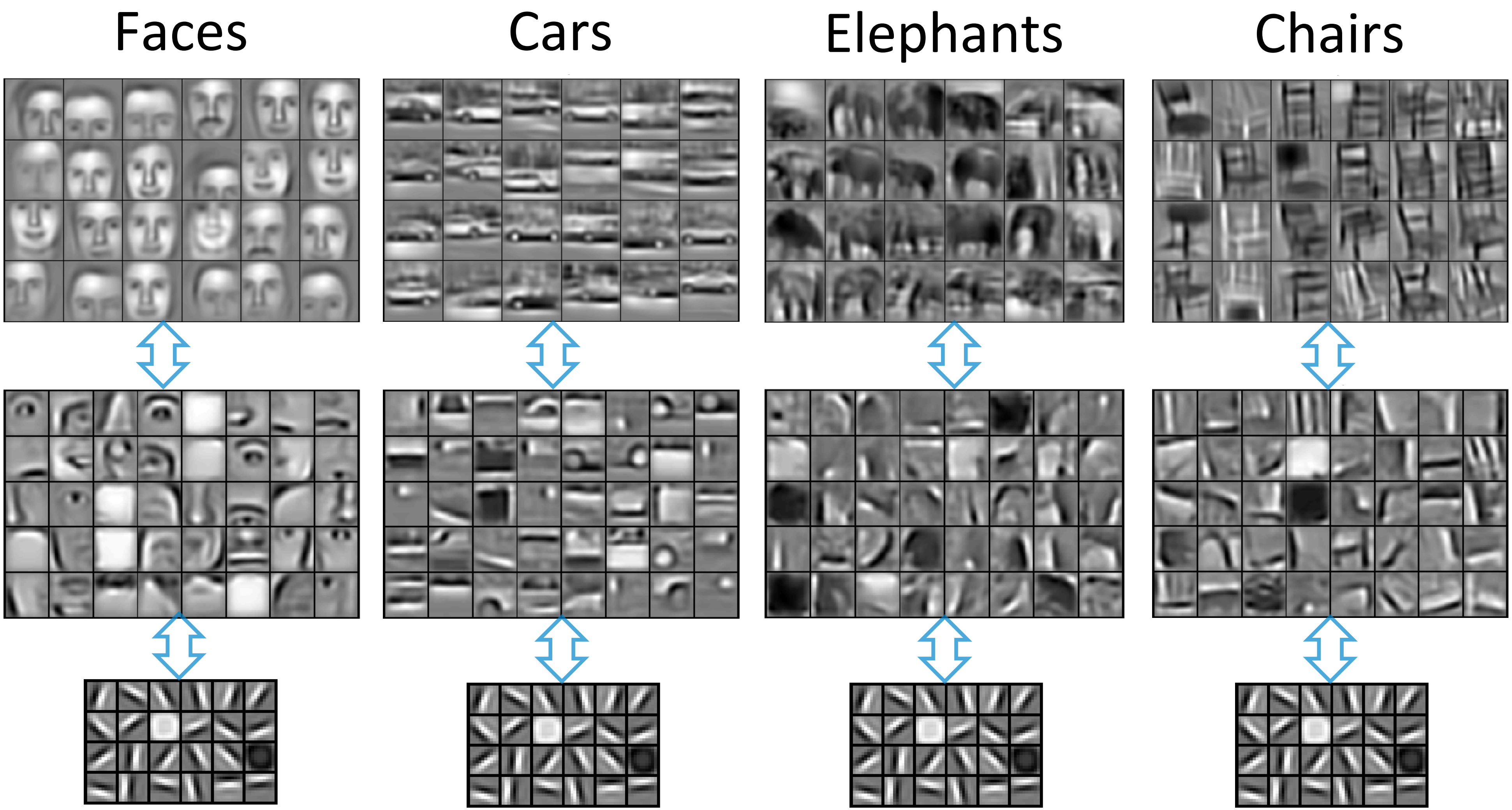
Groups of parts.

Object Parts

Trained on face images.

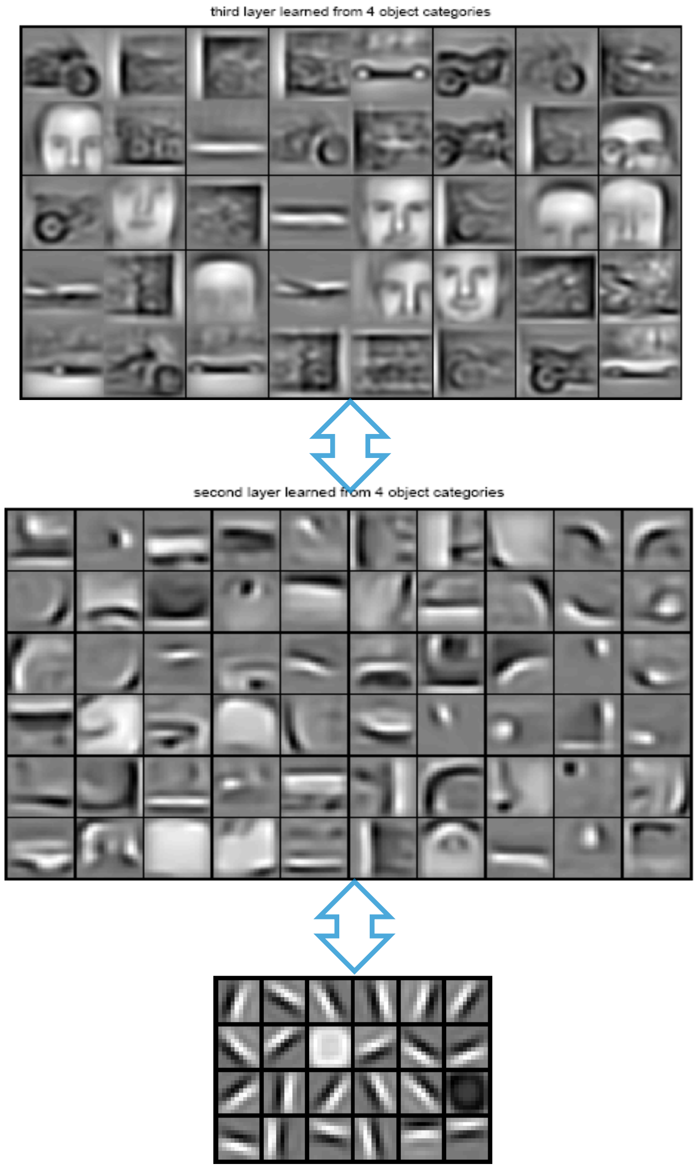
(Lee, Grosse, Ranganath, Ng, ICML 2009)

Learning Part-based Representation



(Lee, Grosse, Ranganath, Ng, ICML 2009)

Learning Part-based Representation



Groups of parts.

Class-specific object parts

Trained from multiple classes (cars, faces, motorbikes, airplanes).

(Lee, Grosse, Ranganath, Ng, ICML 2009)